

SINGLE-LEG HOP DISTANCE PER LAUNCHED ATTEMPT: LANDING QUALIFICATION AND EXACT EIGHT-HOP CALCULATIONS

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ABSTRACT

To determine how retaining failed landings in an eight-hop denominator changes the distance outcome, its variance, and the exposure needed to collect accepted hops. A launched hop contributed limb-length-normalized distance when it satisfied a two-second landing rule; an observed landing failure contributed zero qualified distance. Expectation, variance, and distance-exceedance identities were evaluated with two eight-outcome probability laws and 30 finite laws spanning five validity probabilities, three conditional means, and two distance shapes. Serial dependence, position-specific probabilities, and binary classification errors were evaluated by exact calculations. No participants were tested. Qualified distance equalled validity probability multiplied by mean valid-hop distance. Distribution A had seven valid outcomes with a conditional mean of 150% of limb length and a qualified mean of 131.25%; B had four valid outcomes with corresponding means of 180% and 90.00%. Collecting three accepted hops required 3.43 and 6.00 attempts on average under independent, stationary sampling. Lag-one correlation of 0.6 increased eight-hop mean variance by a factor of 3.0782. Opposing and aligned position trends yielded qualified means of 110.29 and 113.71 despite equal position-average components. Reclassifying A's three longest valid outcomes as failures reduced its mean to 73.25% and reversed the ranking against B. The launched-attempt denominator distinguishes distance among valid landings from qualified distance per hop. Interpretation depends on the acceptance rule, exposure count, and observation status.

Keywords: single-leg hop; landing qualification; eight-hop denominator; classification error; limb comparison; serial dependence.

INTRODUCTION

Single-leg hop testing measures forward displacement while requiring the performer to complete a one-leg landing. Prospective research has examined hop performance in relation to later patient-rated knee function, and reliability studies have included horizontal hopping in lower-limb test batteries (Logerstedt et al., 2012; Kockum and Heijne, 2015). A distance calculated from accepted hops is conditional on satisfying the landing rule. It cannot, by itself, describe how often a launched attempt completes that rule.

The acceptance procedure determines the available denominator. Nilsgård and colleagues repeated unsuccessful hops until two approved attempts were obtained and did not count the failed attempts (Nilsgård et al., 2023). Judd and Sharp examined maximum and repeated single-hop performance, including failed attempts (Judd and Sharp, 2022). Weber and colleagues assessed hop movement quality separately from conventional performance outcomes (Weber et al., 2024). Accepted distance, landing validity, and movement quality therefore correspond to different recorded aspects of the task.

Mechanical interpretation also requires this distinction. Hip, knee, and ankle contributions differ between horizontal and vertical hopping (Kotsifaki et al., 2021), while symmetric hop distance after anterior cruciate ligament reconstruction can

coexist with asymmetric lower-limb biomechanics (Kotsifaki et al., 2022). Completing a landing hold does not quantify joint loading. The analysis below concerns expected distance satisfying a specified landing rule per launched attempt. It derives that expectation from landing validity and accepted distance, then quantifies ranking, exposure, classification, and dependence effects for explicit finite probability laws.

MEASUREMENT DEFINITION

LAUNCH, LANDING QUALIFICATION, AND DISTANCE

Index the participant by i , stance limb by ℓ , and launched position by k . The stance foot leaving the support surface starts an attempt. Hesitation or withdrawal before take-off is a non-launch event. Let $V_{i\ell k} = 1$ denote a valid landing and $V_{i\ell k} = 0$ an observed landing failure.

First landing contact starts a two-second qualification interval. Validity requires support on the tested limb throughout this interval without contralateral-foot contact, hand or external support, or a second airborne phase. Arms and trunk are unrestricted. Support contact before two seconds invalidates the attempt; contact after the interval does not alter its classification. Two-second holds occur in hop assessment (Weber et al., 2024); arm instructions must be fixed because different arm procedures can disagree (Ageberg and Cronström, 2018).

Sagittal rolling from initial toe or heel contact to foot-flat support is permitted if the initially contacting sole point remains within 10 mm of its initial floor location. Translation beyond 10 mm or horizontal shoe-axis rotation beyond 5° invalidates the attempt. These tolerances define the acceptance rule; their physiological validity and examiner agreement have not been estimated. A second airborne phase is a failure even when the foot returns to the same location. Unresolved contact-point or orientation tracking leaves validity unknown until adjudication.

Let $h_{i\ell k}$ be the signed forward displacement from the take-off toe line to the vertical floor projection of the posterior shoe-heel landmark at the first landing-contact frame. Distance is measured along the instructed hop direction. Set $H_{i\ell k} = \max(0, h_{i\ell k})$. A heel behind the line contributes no positive forward distance; the signed displacement remains available. Calibrated camera views determine the heel projection even when the heel is above the floor at first contact. The endpoint is fixed at first contact so later foot motion cannot lengthen the distance. For stance-limb length $L_{i\ell}$, measured from the anterior superior iliac spine to the medial malleolus,

$$D_{i\ell k} = 100 \frac{H_{i\ell k}}{L_{i\ell}}. \quad (1)$$

The unit is percentage of stance-limb length. Toe-to-heel measurement is used in hop testing (Ageberg and Cronström, 2018); first-contact projection fixes the endpoint for this analysis. Centimetres and limb length remain necessary because division by limb length does not establish removal of anthropometric effects.

THE EIGHT-HOP DENOMINATOR

The qualified outcome is

$$S_{i\ell k} = \begin{cases} D_{i\ell k}, & V_{i\ell k} = 1, \\ 0, & V_{i\ell k} = 0 \text{ and landing} \\ & \text{failure is observed.} \end{cases} \quad (2)$$

Zero means that the landing produced no distance satisfying the complete task. It does not describe physical displacement or failure severity. A known failed landing therefore needs no distance value to assign $S = 0$. Unresolved validity, a lost recording, or a missing distance after a valid landing produces an unknown outcome.

For B launched attempts with every outcome resolved,

$$\hat{\theta}_{i\ell} = \frac{1}{B} \sum_{k=1}^B S_{i\ell k}. \quad (3)$$

and the position-average expectation is

$$\theta_{i\ell, B} = \frac{1}{B} \sum_{k=1}^B E(S_{i\ell k}). \quad (4)$$

The acquisition definition uses $B = 8$ for each limb. Practice, fatigue, or strategy can change $E(S_k)$ across positions without invalidating the average in Eq. (4). Linearity of expectation gives $E(\hat{\theta}) = \theta_B$ under complete observation of that block. Independence is unnecessary for this identity. Selection of participants who finish the block changes the population to which the expectation applies.

ACQUISITION SETTINGS AND EVENT RECORDS

Table 1 specifies preparation, timing, cameras, and decision tolerances. The eligibility definition comprises physically active adults aged 18–40 years able to perform single-leg hopping; hopping restrictions, acute symptoms, and unresolved instability exclude participation. No performance, safety, or reliability estimates were obtained for this population.

TABLE 1. ACQUISITION SETTINGS FOR AN EIGHT-HOP BLOCK PER LIMB. THE SETTINGS DEFINE THE TASK AND RECORDING PROCEDURE; NO PARTICIPANTS WERE TESTED IN THIS MATHEMATICAL INVESTIGATION.

Item	Specification and retained record
Preparation	Five minutes of cycling; two submaximal and two maximal practice hops per limb. Retain practice outcomes separately.
Measurement exposure	Eight launched hops per limb; randomized left-first or right-first blocks. Failed landings consume an attempt.
Instruction	"Hop as far forward as you can and maintain the specified landing for two seconds." Arms free; no external support. Same footwear and level, nonslip surface across sessions.
Recovery	Target 45 s between take-offs and 3 min between limb blocks; retain actual times-tamps and departures from the schedule.
Recording	Synchronized frontal and sagittal views at 120 frames/s or faster, with camera calibration for three-dimensional reconstruction and a marked floor plane. Retain frame rate, synchronization error, calibration residuals, and views.
Distance endpoint	Posterior heel landmark projected onto the floor at first contact. Use a calibrated floor reference and record to 0.5 cm; confirm endpoint error with known-distance checks. Recording resolution does not establish measurement accuracy.
Limb length	Two measurements to 0.5 cm; if they differ by more than 1 cm, obtain a third and use the median of three. Store individual measurements.
Landing classification	Two-second interval; permitted sagittal rolling; 10 mm translation and 5° horizontal rotation tolerances. Two raters independently classify masked attempt identifiers before adjudication.
Exceptions	Record non-launch, recording loss, unresolved validity, support contact, relocation, additional hop, and safety termination as distinct events. Multiple failure codes are permitted.

A non-launch event retains the same launched-position index and its elapsed time. Recording loss after launch consumes that position and leaves an unknown outcome. A failed landing consumes its position without replacement. Pain, giving way, or examiner concern ends the block. A block with fewer than eight launches or an unresolved outcome has no fully observed eight-hop mean. Launch completion and outcome resolution are recorded separately.

EXACT PROPERTIES OF QUALIFIED DISTANCE

VALIDITY PROBABILITY AND CONDITIONAL DISTANCE

Select one block position uniformly. Define $p = \Pr(V = 1)$, $\mu = E(D \mid V = 1)$, and $\sigma_D^2 = \text{Var}(D \mid V = 1)$. For $p > 0$,

$$\theta = E(S) = p\mu. \quad (5)$$

At $p = 0$, the qualified expectation is zero and the conditional mean is undefined. Accepted-hop distance answers a conditional question; θ answers a per-launch question. Replacing failures is compatible with a conditional target, but it removes failure frequency from a distance mean unless that frequency is retained separately.

For two laws A and B, the qualified mean of A exceeds that of B exactly when

$$p_A \mu_A > p_B \mu_B. \quad (6)$$

If $\mu_A < \mu_B$, the conditional ranking reverses when $p_A/p_B > \mu_B/\mu_A$. Probability and conditional distance must therefore be available to interpret the ranking.

PROBABILITY OF A QUALIFIED HOP BEYOND A DISTANCE

For $c \geq 0$, define

$$q(c) = \Pr(V = 1, D > c). \quad (7)$$

The sample estimate divides the number of valid hops with $D > c$ by all B launched attempts. For a nonnegative qualified outcome with finite expectation,

$$\theta = \int_0^\infty q(c) dc. \quad (8)$$

This follows from $S = \int_0^\infty \mathbf{1}(S > c) dc$ and integration of the expected indicator. The area weights every unit of qualified distance equally. A prespecified threshold instead measures the probability of completing a particular distance demand. Crossing curves can give different rankings at different thresholds.

VARIANCE AND DEPENDENCE BETWEEN ATTEMPTS

For a stationary attempt law with $p > 0$,

$$\text{Var}(S) = p\sigma_D^2 + p(1-p)\mu^2. \quad (9)$$

At $p = 0$, the score is identically zero and its variance is zero. For $p > 0$, the first term represents valid-distance variation and the second represents alternation between validity and failure. For a block,

$$\text{Var}(\hat{\theta}) = \frac{1}{B^2} \left[\sum_{k=1}^B \text{Var}(S_k) + 2 \sum_{k < r} \text{Cov}(S_k, S_r) \right]. \quad (10)$$

Equal marginal variances and zero pairwise covariances reduce this expression to $\text{Var}(S)/B$; independent, identically distributed attempts are sufficient. Constant pairwise correlation ρ gives the multiplier $1 + (B-1)\rho$. With eight attempts and $\rho = 0.2$, that multiplier is 2.4. Sampling further attempts from one person cannot replace sampling independent participants (Aarts et al., 2014).

EXPOSURE REQUIRED FOR ACCEPTED HOPS

Under independent, identically distributed attempts with constant $p > 0$, let T_m be the number of launches required for m valid landings. Then

$$E(T_m) = \frac{m}{p}, \quad \text{Var}(T_m) = \frac{m(1-p)}{p^2}. \quad (11)$$

and

$$\Pr(T_m \leq b) = \sum_{j=m}^b \binom{b}{j} p^j (1-p)^{b-j}. \quad (12)$$

Accepted distances follow $D \mid V = 1$. Their maximum is an accepted-distance order statistic, while exposure depends on p . Position-dependent success invalidates the stationary replacement formulas. Repeated-hop performance warrants retaining the number and order of all launches (Judd and Sharp, 2022).

FINITE-LAW CALCULATIONS

EIGHT-OUTCOME LAWS AND ARITHMETIC

Table 2 defines probability laws A and B. Each row has probability $1/8$. F denotes an invalid landing with qualified outcome zero; physical distance is unspecified for that row. Row numbers identify outcomes, not temporal athlete records. The numerical investigation uses these finite laws without participant measurements.

TABLE 2. EIGHT-OUTCOME PROBABILITY LAWS. NUMERICAL ENTRIES ARE VALID-HOP DISTANCES IN PERCENTAGE OF STANCE-LIMB LENGTH. F HAS QUALIFIED OUTCOME ZERO. EACH ROW HAS PROBABILITY $1/8$ IN ITS RESPECTIVE LAW.

Atom	Distribution A	Distribution B
1	142	176
2	150	F
3	154	184
4	148	F
5	152	180
6	146	F
7	158	F
8	F	180

Means and variances follow from weighted sums. The binomial sum in Eq. (12) gives accepted-hop collection probabilities. Enumerating all 7^3 ordered valid triples for A and 4^3 for B gives the expected maximum of three accepted distances. Rectangle integration between distance knots gives the tail area. No random sampling enters these calculations.

OPPOSITE DISTANCE RANKINGS AND UNEQUAL EXPOSURE

A's seven valid distances sum to 1050. Hence $p_A = 0.875$, $\mu_A = 150$, and $\theta_A = 1050/8 = 131.25$. B's four valid distances sum to 720, giving $p_B = 0.5$, $\mu_B = 180$, and $\theta_B = 90$. B exceeds A by 30 conditional-distance percentage points, whereas A exceeds B by 41.25 qualified-distance percentage points (Table 3).

TABLE 3. EXACT QUANTITIES FOR A AND B. REPLACEMENT QUANTITIES USE INDEPENDENT STATIONARY DRAWS UNTIL THREE VALID LANDINGS. DISTANCE MEANS ARE PERCENTAGES OF LIMB LENGTH AND VARIANCES ARE SQUARED PERCENTAGE POINTS.

Quantity	A	B
Valid-landing probability, p	0.875	0.500
Mean valid-hop distance, μ (% limb length)	150.00	180.00
Landing-qualified distance, θ (% limb length)	131.25	90.00
Conditional distance variance (squared percentage points)	24.00	8.00
Attempt-outcome variance (squared percentage points)	2481.94	8104.00
Expected attempts to obtain 3 valid landings	3.43	6.00
Probability of obtaining 3 valid landings within 8 attempts	0.999915	0.855469
Expected maximum of 3 valid hops (% limb length)	154.16	182.25

B's valid-distance variance is 8 rather than A's 24, but its qualified variance is 8104 rather than 2481.9375. Failure frequency explains the larger per-launch variance. Collecting three valid hops requires 6.00 launches on average for B and 3.428571 for A, a 75% exposure difference.

Below 142% of limb length, the exceedance probabilities are 0.875 for A and 0.500 for B. Above A's maximum of 158%, only B can exceed the threshold, up to B's maximum of 184%. The integrated mean favours A, while a distance threshold beyond 158% favours B when it is below 184%.

Within eight independent launches, B accumulates three valid landings with probability 0.855469. Its non-completion probability for that quota is 0.144531. A's collection probability is 0.999915 (Figure 2). Selecting only blocks that satisfy the quota selects on landing success.

DISTANCE SHAPE AND THE RANKING BOUNDARY

With $p_A = 0.875$ and $\mu_A = 150$, the equality condition is $p_B(\mu_B/\mu_A) = 0.875$. At $\mu_B = 180$, the boundary is $p_B = 0.729167$. Figure 3 gives the full boundary as B's probability and mean-distance ratio vary.

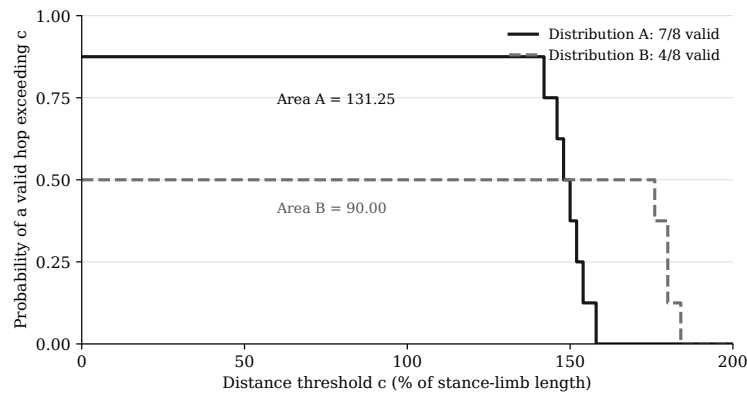


FIGURE 1. QUALIFIED-DISTANCE EXCEEDANCE PROBABILITIES FOR LAWS A AND B: $q(c) = \Pr(V = 1, D > c)$. THE AREAS ARE 131.25 AND 90.00 PERCENTAGE POINTS OF LIMB LENGTH. STRICT EXCEEDANCE DETERMINES THE VALUES AT DISTANCE KNOTS.

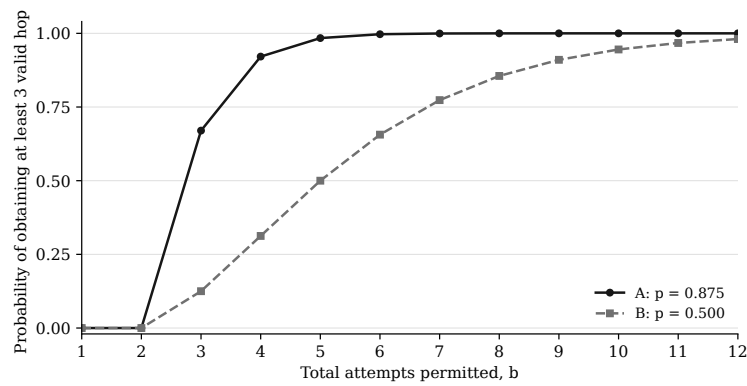


FIGURE 2. PROBABILITY OF AT LEAST THREE VALID LANDINGS WITHIN b LAUNCHES FOR $p = 0.875$ AND $p = 0.500$. POINTS ARE EXACT BINOMIAL PROBABILITIES; CONNECTING LINES LINK INTEGER BUDGETS. THE CALCULATION USES INDEPENDENT STATIONARY ATTEMPTS.

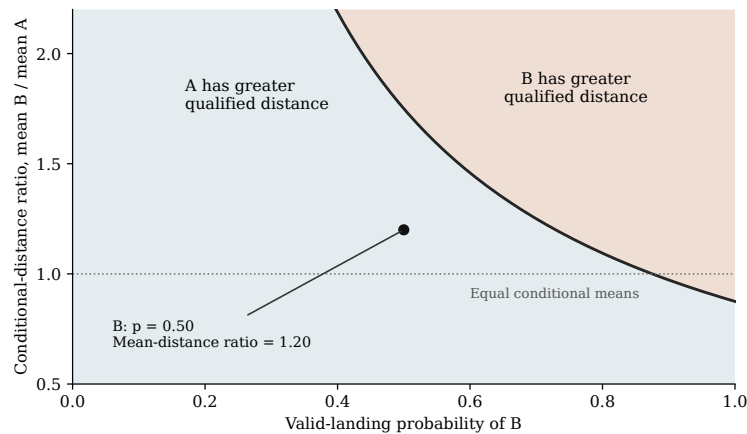


FIGURE 3. EQUALITY OF QUALIFIED MEANS AT $p_B(\mu_B/\mu_A) = 0.875$, WITH A FIXED AT $p_A = 0.875$. SHADING IDENTIFIES THE GREATER QUALIFIED MEAN. THE DOTTED LINE DENOTES EQUAL CONDITIONAL MEANS; B'S POINT (0.50, 1.20) LIES ABOVE THAT LINE BUT ON A'S SIDE OF THE QUALIFIED-MEAN BOUNDARY.

The 30 finite laws use $p \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$, $\mu \in \{100, 150, 200\}$, and two conditional shapes. Equal masses at $(0.8\mu, 1.2\mu)$ give variance $0.04\mu^2$. Masses $(0.9, 0.1)$ at $(0.9\mu, 1.9\mu)$ give the same mean and variance $0.09\mu^2$. An atom of mass $1 - p$ at qualified zero completes each law. Direct summation satisfies Eqs. (5), (8), and (9) in all 30 laws. Qualified means range from 20 to 200% of limb length, and independent eight-hop standard errors range from 7.071068 to 38.340579 percentage points. Distance shape changes precision without changing the mean at fixed p and μ .

CLASSIFICATION ERROR AND CHANGES IN ACCEPTANCE

Let $\alpha = \Pr(V^* = 0 \mid V = 1)$ and $\beta = \Pr(V^* = 1 \mid V = 0)$ denote false-negative and false-positive rates against a reference classification. For the classification calculation, assign a physical normalized distance D^{phys} to every outcome and let $\nu = E(D^{\text{phys}} \mid V = 0)$. Conditional independence between distance and classification error within each validity class gives

$$\theta^* = (1 - \alpha)p\mu + \beta(1 - p)\nu. \quad (13)$$

False negatives remove qualified distance; false positives add physical distance from invalid landings. Calculating the second term requires failed-hop distance information that the qualified score itself does not require. Assigning the failed outcomes physical distance 150 in A and 180 in B sets $\nu_A = 150$ and $\nu_B = 180$ for Table 4. These are specified mathematical values, not measured displacements.

TABLE 4. QUALIFIED MEANS AFTER CLASSIFICATION ERRORS, WITH $\nu_A = 150$ AND $\nu_B = 180$. THE SAME α AND β APPLY TO BOTH LAWS. ENTRIES ARE PERCENTAGES OF LIMB LENGTH; ERROR RATES ARE FIXED CALCULATION INPUTS.

False-negative rate α	False-positive rate β	θ_A^*	θ_B^*
0.00	0.00	131.25	90.00
0.05	0.00	124.69	85.50
0.00	0.05	132.19	94.50
0.10	0.10	120.00	90.00

Changing the acceptance rule changes V itself. Reclassifying A's longest one, two, or three valid outcomes as failures gives means of 111.50, 92.25, and 73.25, respectively, while B remains at 90.00. Reclassifying three reverses the ranking. No hold times or foot trajectories accompany the finite laws, so these values cannot identify the effect of a particular translation, rotation, or hold-time threshold on athletes.

SERIAL DEPENDENCE AND ATTEMPT POSITION

The first score follows the eight-outcome law. Each subsequent position retains the preceding outcome with probability r and otherwise draws independently. The transition matrix is $P = rI + (1 - r)\mathbf{1}\mathbf{1}^T/8$. Marginal probabilities remain unchanged, and lag- h score correlation is r^h for a nondegenerate law. Therefore,

$$\frac{\text{Var}(\hat{\theta})}{\text{Var}(S)/B} = 1 + 2 \sum_{h=1}^{B-1} \left(1 - \frac{h}{B}\right) r^h. \quad (14)$$

Direct transition-matrix covariance calculations give the same variance. With $r = 0.6$ and $B = 8$, the multiplier is 3.0782464 (Table 5). This transition law represents score persistence; it does not estimate an athlete's fatigue response.

TABLE 5. EIGHT-HOP MEAN PRECISION UNDER THE TRANSITION MATRIX $P = rI + (1 - r)\mathbf{1}\mathbf{1}^T/8$. STANDARD ERRORS ARE PERCENTAGE POINTS OF LIMB LENGTH; THE MULTIPLIER IS RELATIVE TO INDEPENDENT ATTEMPTS.

Lag-one correlation r	Variance multiplier	SE for A	SE for B
0.0	1.0000	17.61	31.83
0.3	1.7041	22.99	41.55
0.6	3.0782	30.90	55.84

Position-specific probabilities $p_k = 0.9 - 0.4(k - 1)/7$ decline from 0.9 to 0.5. Pairing them with $\mu_k = 140 + 40(k - 1)/7$ gives $\frac{1}{8} \sum_k p_k \mu_k = 110.285714$. Pairing them with $\mu_k = 180 - 40(k - 1)/7$ gives 113.714286. Both position-average component pairs are (0.7, 160), whose unweighted product is 112. Averaging within-position products retains their covariance; multiplying separate position averages loses it. A stationary negative-binomial replacement calculation does not apply to these changing probabilities.

INFERENCE AND INTERPRETATION

COMPONENTS AND PAIRED LIMB CONTRASTS

A complete block supplies B , valid count n_V , $\hat{p} = n_V/B$, valid-distance mean $\hat{\mu}$, and qualified mean $\hat{\theta}$. For $n_V > 0$, $\hat{\theta} = \hat{p}\hat{\mu}$ holds before rounding. For $n_V = 0$, qualified distance is zero and conditional distance is undefined. Failure codes, centimetres, limb length, launch order, and timestamps accompany these quantities.

For injured (I) and comparison (C) limbs with $p_I > 0$ and $\theta_C > 0$,

$$100 \frac{\theta_I}{\theta_C} = \frac{p_I}{p_C} \left(100 \frac{\mu_I}{\mu_C} \right). \quad (15)$$

Using B as I and A as C gives a conditional-distance ratio of 120% and a qualified-distance ratio of 68.571429%. At $p_I = 0$ and $\theta_C > 0$, the direct qualified ratio is zero but the component factorization is unavailable because μ_I is undefined. At $\theta_C = 0$, the direct ratio is undefined. A comparison limb can itself change after injury (Wellsandt et al., 2017); qualified distance does not resolve that comparator problem. Absolute component values and the limb difference remain necessary.

The paired contrast is $\Delta_i = \hat{\theta}_{iI} - \hat{\theta}_{iC}$. A participant bootstrap retains both limbs and all attempts when resampling a person. Eight launches from one person remain one participant cluster (Aarts et al., 2014).

AVERAGING MODEL PREDICTIONS

A logistic mixed model for validity and a conditional-distance model can represent limb, launch position, block order, and participant effects. For covariates x and a prespecified target distribution G , write $p(x) = \Pr(V = 1 \mid x)$ and $\mu(x) = E(D \mid V = 1, x)$. The standardized expectation is

$$\theta_G = \int p(x)\mu(x) dG(x), \quad (16)$$

At $p(x) = 0$, the integrand is zero without assigning a conditional mean. Equal weights on $(p, \mu) = (0.9, 150)$ and $(0.5, 180)$ give 112.5; multiplying the separate weighted means gives 115.5. The standardized conditional mean is $\int p(x)\mu(x)dG(x) / \int p(x)dG(x)$ when validity probability is positive. Joint random effects require integration of their conditional product. A conditional-distance intervention contrast conditions on post-intervention validity and cannot be substituted for an overall qualified-distance contrast.

INCOMPLETE LAUNCHES AND UNRESOLVED OUTCOMES

Let $C_i = 1$ indicate completion of both prescribed launch blocks and $O_i = 1$ resolution of every outcome. Fully observed paired blocks target $E(\Delta_i \mid C_i = 1, O_i = 1)$. That expectation does not automatically apply to all eligible participants. Counts of eligible, enrolled, initiated, completed, and fully observed participants distinguish the analysis populations. Limb-specific counts are necessary when only one block is complete.

Technical missingness within completed blocks requires an observation assumption. Multiple imputation can preserve binary validity, the score's mass at zero, and nonnegative distance conditional on validity while conditioning on limb, position, prior outcomes, and recording circumstances. Its uncertainty must enter the analysis. Missing-at-random cannot be established from the finite laws. Complete-case and assumption-dependent estimates answer different observation conditions and should be identified separately.

Safety termination leaves the remaining task outcomes undefined under a procedure that prohibits further hopping. Assigning zero to unlaunched positions changes the outcome; extrapolation requires a hypothetical continuation condition. Completion status and observed performance therefore remain separate outcomes for the eligible population.

MEANING OF ZERO AND LIMITS OF INTERPRETATION

The score uses $u(V, D) = VD$: distance contributes only after landing qualification. It assigns the same zero to a brief support contact and a major loss of balance, without equating their severity. Separate failure codes and movement-quality assessments retain that distinction (Weber et al., 2024). A utility penalizing each failure by $\lambda \geq 0$ instead has expectation $\theta - \lambda(1 - p)$. The choice of λ expresses an additional task preference rather than a property estimated by the hop score.

Shorter self-selected hops can increase validity probability. The product may reward that strategy, so both components and the exceedance curve are necessary to distinguish distance changes from success-frequency changes. Agreement between raters must address the 10 mm and 5° decisions as well as support contacts and additional hops. Test-retest agreement concerns the entire eight-hop mean under fixed timing, footwear, and familiarization; it cannot be inferred from the distance dispersion of accepted hops alone.

Qualified distance is determined by p and μ and contributes no independent measurement information beyond their joint values. Predictive improvement over an additive model of the components can reflect the omitted product interaction. The exact calculations establish neither injury prediction nor return-to-sport clearance. Prospective reinjury research concerns actual rehabilitation and sport exposure (Grindem et al., 2016); clinical thresholds, responsiveness, and participant-level measurement agreement require participant observations.

CONCLUSION

An eight-hop mean that retains failed landings measures qualified distance per launched attempt. In the specified laws, it ranks A at 131.25% above B at 90.00%, despite conditional means of 150% and 180%. The denominator also exposes a 75% difference in expected launches needed for three accepted hops. Classification, serial dependence, and within-block component association change the score or its precision through identifiable mathematical terms. Validity probability, conditional distance, qualified distance, and completion status must remain available for interpretation.

DECLARATIONS

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest related to this study.

DATA AVAILABILITY

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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