

LATE-TRAINING ALLOCATION IN ADOLESCENT COMMUNITY SPORT: CAPACITY, CONCENTRATION AND WEEKLY ROTATION

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ABSTRACT

To determine the minimum number of late training sessions, limit the largest squad-level allocation and balance the remaining late appointments within a shared youth-sport calendar. Nine squads shared two courts from Tuesday to Friday, with 32 weekly 90-minute appointments starting at 16:00, 17:30, 19:00 and 20:30. Each squad required three sessions on distinct days; squads A–C were excluded from 20:30 starts. Integer optimization minimized total late sessions, maximum late sessions per eligible squad, pairwise differences in late-session counts and then summed start-time span. Ten operating conditions, all 24 individual non-late court-slot closures, equal-count quotas and a six-week rotation were evaluated. The intact calendar required three late sessions. Assigning all three to I gave a maximum of three and a Gini coefficient of 0.833; distributing one each to E, G and I gave a maximum of one and a Gini coefficient of 0.500. Four specified non-late closures required seven late sessions and a maximum of two. The vectors (1, 2, 0, 2, 2, 0) and (1, 2, 1, 1, 1, 1) both attained these values, but their Gini coefficients were 0.405 and 0.119. Every individual non-late closure permitted four late sessions with a maximum of one. Restricting H and I to 20:30 forced six late sessions and a maximum of three; one late session per eligible squad was infeasible. Alternating D–F and G–I over six intact weeks gave every eligible squad three late sessions. Minimizing the largest late-session count leaves the distribution among the other squads undetermined. Pairwise-difference minimization and weekly rotation balanced late appointments while retaining all requested practices. The results concern the specified scheduling restrictions and do not quantify sleep, injury or participation outcomes.

Keywords: sports management; adolescent sport; facility allocation; integer programming; distributional equity; training schedules; weekly rotation.

INTRODUCTION

An adolescent training calendar allocates both court access and finishing times. A 90-minute practice beginning at 20:30 ends at 22:00, whereas one beginning at 19:00 ends at 20:30. Two timetables can occupy the same number of court appointments while assigning these late finishes to different squads. Sports-scheduling research addresses venue allocation and rest differences between competitors (Ribeiro, 2012; Çavdaroğlu and Atan, 2020); recurring practice allocation also requires decisions about which squads receive the last evening booking.

Access to community-sport spaces can remain unequal despite formal inclusion policies (Jeanes et al., 2021), and the geographical distribution of facilities can differ across social groups (Shen et al., 2020). Within a shared venue, booking rules determine how squads divide the available appointments. Limiting the largest late-session count protects the most exposed squad, but does not determine the counts assigned to every other eligible squad. Task-assignment research distinguishes this maximum-based criterion from lexicographic ordering of the full burden distribution (Ding and Balakrishnan, 2023).

Training times also intersect with sleep opportunity. Movement guidelines include age-appropriate sleep in daily activity recommendations (Tremblay et al., 2016), and athlete sleep guidance considers training, travel and individual schedules (Walsh et al., 2021). Saidi and colleagues examined chronotype-dependent responses to afternoon and evening high-intensity exercise in adolescent athletes (Saidi et al., 2023). Brown and colleagues found differences in sleep timing across academy-football training schedules alongside generally resilient sleep behavior (Brown et al., 2020). Studies of adolescent swimmers and youth footballers examined associations between training schedules and sleep (Gudmundsdottir, 2020; Whitworth-Turner et al., 2019); early-morning training advanced wake times without corresponding bedtime changes in competitive adolescent athletes (Steenekamp et al., 2021). These findings justify considering practice timing alongside practice volume, while the consequences of a particular 20:30 booking depend on the athletes' circumstances.

This study determines exact allocation limits for nine squads requesting 27 sessions from a 32-appointment calendar. It tests whether the six late-eligible squads can share unavoidable 22:00 finishes, how one and four non-late closures change that distribution, and whether equal-count quotas remain feasible under coach and transport restrictions. Pairwise exposure differences distinguish allocations with the same maximum; a six-week calculation tests accumulated exposure when weekly quota groups alternate.

METHODS

COURT CALENDAR AND OPERATING RESTRICTIONS

Nine hypothetical squads, A–I, share north (N) and south (S) courts on Tuesday–Friday. Each court supplies four non-overlapping 90-minute appointments starting at 16:00, 17:30, 19:00 and 20:30. The last appointment ends at 22:00. Weekly supply is $2 \times 4 \times 4 = 32$ appointments, or 48 court-hours. Every squad requires exactly three sessions on distinct days, giving 27 requested appointments and 40.5 court-hours. The study calculates feasible allocations for this specified calendar.

A–C are designated under-14 squads; D–I are designated under-16 or under-17 squads. A stipulated safeguarding restriction excludes A–C from 20:30 starts, leaving six late-eligible squads. These age categories define eligibility in the calculation; the 20:30 restriction is an administrative assumption. Courts are interchangeable in quality and function. The calendar includes no setup interval, shared athlete roster or additional coach-travel time. “Non-late” denotes a start before 20:30, including 19:00.

Coach-sharing pairs A/B, D/E and G/H cannot train simultaneously. Transport exclusions prohibit A and B from Tuesday 16:00, D and E from Thursday 16:00, and G and H from Wednesday 16:00, on either court. These restrictions are fixed inputs. Session counts represent booked practice opportunities; physical intensity and attendance are outside the allocation variables.

ALLOCATION VARIABLES AND FEASIBILITY

Let i index squads, $d \in \{1, 2, 3, 4\}$ days, $c \in \{N, S\}$ courts and $t \in \{1, 2, 3, 4\}$ start-time bands. Define $x_{idct} \in \{0, 1\}$ as receipt of an appointment. If squad i requires r_i sessions and a_{dct} indicates an available appointment, the constraints are

$$\sum_{d,c,t} x_{idct} = r_i \quad \forall i, \quad (1)$$

$$\sum_{c,t} x_{idct} \leq 1 \quad \forall i, d, \quad (2)$$

$$\sum_i x_{idct} \leq a_{dct} \quad \forall d, c, t. \quad (3)$$

For every shared-coach pair (u, v) ,

$$\sum_c (x_{udct} + x_{vdct}) \leq 1 \quad \forall d, t. \quad (4)$$

Junior, transport and other availability exclusions fix the relevant variables to zero. These restrictions define the feasible set $\mathcal{F}$.

The implementation enumerates every allowed weekly day/start-time pattern for each squad and selects one pattern per squad. Pattern occupancy is limited to $\sum_c a_{dct}$ at each (d, t) ; shared-coach occupancy is limited to one. This is equivalent to Eqs. (1)–(4) for the present interchangeable courts: any selected patterns can be assigned to the available

physical courts independently at each day/start-time. The equivalence would fail if squads had court-specific eligibility restrictions. A separate appointment-level checker validates every decoded timetable.

ORDERED EXPOSURE AND REGULARITY OBJECTIVES

Define the late-session count $L_i = \sum_{d,c} x_{idcA}$ and eligible set $\mathcal{O}$. The ordered objectives are

$$Z^* = \min_{x \in \mathcal{F}} \sum_i L_i, \quad (5)$$

$$M^* = \min_{x \in \mathcal{F}, \sum_i L_i = Z^*} \max_{i \in \mathcal{O}} L_i, \quad (6)$$

$$T^* = \min_{x \in \mathcal{F}, \sum_i L_i = Z^*, L_i \leq M^*} \sum_{\substack{i < j \\ i, j \in \mathcal{O}}} |L_i - L_j|. \quad (7)$$

Each optimum is fixed before the next objective is solved. The first stage fixes the smallest feasible total of late appointments. The second fixes the smallest feasible maximum among eligible squads. The third minimizes the sum of pairwise differences subject to both preceding optima. This three-stage ordering determines total burden, largest burden and residual concentration; pairwise-difference minimization differs from lexicographic ordering of the sorted exposure vector.

The maximum is represented by an integer m with $L_i \leq m$. Absolute differences use integer variables $u_{ij} \geq L_i - L_j$ and $u_{ij} \geq L_j - L_i$. On the three principal calendars, a fourth objective minimizes $S = \sum_i R_i$, where R_i is the largest minus the smallest start-time band in that squad's selected pattern. One span unit is a 90-minute start-time difference. S measures within-squad variation in booked start times.

CAPACITY BOUNDS AND EXPOSURE DISTRIBUTION

For demand $D = \sum_i r_i$ and E available non-late appointments,

$$Z^* \geq \max(0, D - E), \quad M^* \geq \lceil Z^*/n \rceil, \quad n = |\mathcal{O}|. \quad (8)$$

Attainment requires a timetable satisfying the daily, court, coach and eligibility restrictions. With $Z = nq + s$, $0 \leq s < n$, the least dispersed unrestricted integer vector contains s entries of $q + 1$ and $n - s$ entries of q . Its pairwise difference sum is

$$T \geq s(n - s). \quad (9)$$

For unrestricted nonnegative integer counts, transferring one count from an entry at least two larger than another to that smaller entry cannot increase dispersion; repeated transfers yield this adjacent-integer form. Thus a feasible vector attaining Equation (9) certifies the concentration minimum. Availability or individual session limits can prevent attainment.

The eligible-squad Gini coefficient is $G = T/(nZ)$ when $Z > 0$; $G = 0$ is adopted by convention when $Z = 0$. Because n and Z are fixed during stage three, minimizing T minimizes G . The junior exclusion is not counted as inequality within the eligible cohort. The eligible cohort must accompany the coefficient: a single eligible squad has $G = 0$ even when it receives every late session. Utilization is $D / \sum_{d,c,t} a_{dct}$ and describes occupied usable capacity.

COURT CLOSURES, DEMAND CHANGES AND ALLOCATION RULES

The principal calendars are: intact supply; closure of Thursday 19:00 S; and that closure together with Tuesday 16:00 N, Friday 17:30 S and Wednesday 19:00 N. These remove zero, one and four non-late appointments, respectively. The calculation also removes each of the 24 non-late court appointments individually. The four-closure condition uses the four appointments listed above.

Seven additional settings vary one element of the intact case: eight squads A–H; ten squads A–J; two sessions per squad; five junior squads A–E; eight junior squads A–H; squads H and I available only at 20:30; and four sessions per squad. Other

restrictions retain their specified indices. J has no extra transport or shared-coach exclusion. Changing the number of junior squads changes only late-start eligibility; squad session requirements remain unchanged.

Three allocation rules isolate the effect of each decision. Incumbent priority restricts all late starts to I. The maximum-cap rule retains Eqs. (5)–(6) and leaves T unrestricted; its displayed timetable assigns late counts (1, 2, 0, 2, 2, 0) to D–I. A cyclic-quota rule calculates $q = \lfloor Z^*/n \rfloor$ and gives $q + 1$ late sessions to the first $s = Z^* \bmod n$ eligible squads, and q to the rest. The exact feasibility solver then checks those quotas. The quota rule uses Z^* from the first optimization stage, then fixes each eligible squad's late count and solves the remaining appointment constraints.

For a six-week illustration, two intact-calendar quota groups alternate. D–F receive one late appointment each in odd weeks; G–I receive one each in even weeks. The two weekly timetables are solved and checked. Repeating D–F in every week provides the fixed-group comparison. Weekly availability and session requirements remain unchanged throughout the six-week calculation.

RESULTS

EXPOSURE PROTECTION AND REMAINING CONCENTRATION

The intact calendar had 24 non-late appointments and 27 requested sessions. Both incumbent priority and balanced allocation attained the exact total-late bound of three and utilization of $27/32 = 84.4\%$. Incumbent priority assigned all three to I, producing a maximum of three and $G = 0.833$. Balanced allocation assigned one late session to each of three eligible squads, giving a maximum of one, $T = 9$ and $G = 0.500$. Zero weekly Gini is impossible for three indivisible late sessions distributed over six eligible squads.

Table 1 lists the certified principal outcomes. Every timetable delivers 27 sessions; incumbent priority assigns every late appointment to I.

TABLE 1. PRINCIPAL ALLOCATION OUTCOMES. EXPOSURE OBJECTIVES AND, WHERE SHOWN, START-TIME SPAN ARE CERTIFIED OPTIMA UNDER THE INDICATED RULE.

Allocation	Usable	Late Z	Max. M	T	Gini	Span S
Intact; incumbent priority	32	3	3	15	0.833	2
Intact; balanced	32	3	1	9	0.500	5
One closure; balanced	31	4	1	8	0.333	6
Four closures; cap-only witness	28	7	2	17	0.405	–
Four closures; balanced	28	7	2	5	0.119	10

T is the sum of pairwise eligible-squad exposure differences. The cap-only row is a selected feasible witness with optimal Z and M ; its concentration and span are not optimized. Utilization is 84.4%, 87.1% and 96.4% for zero, one and four closures, respectively.

Four closures left 20 non-late appointments and forced seven late sessions. A one-session cap is impossible because six eligible squads cannot absorb seven appointments under that cap. However, maximum protection at two does not select a unique distribution. The concentrated vector (1, 2, 0, 2, 2, 0) has $T = 17$; the balanced vector contains five ones and one two and has $T = 5$. Its Gini is lower by $12/42 = 0.286$, without changing total late appointments, maximum exposure or delivered sessions (Figure 1). Both complete timetables satisfy the court, daily-session, coach, transport and junior restrictions.

The adjacent-integer concentration bound was attained in all three principal calendars. Once exposure was fixed, the exact minimum summed start-time span was two bands under incumbent priority and five under intact balanced allocation. Balancing exposure therefore increased the minimum summed span by three bands, equivalent to 270 minutes summed across squads. This value describes appointment times.

CLOSURE LOCATION, DEMAND AND ELIGIBILITY

Every one of the 24 possible individual non-late closures remained feasible and attained $Z = 4$, $M = 1$ and $T = 8$ ($G = 0.333$). The optimal exposure values were identical for all 24 individually removed appointments. Multiple-closure combinations were evaluated only for the four specified removals.

Eight squads or two sessions per squad allowed all demand to be placed before 20:30. Ten squads generated 30 requested sessions and six unavoidable late appointments, with a maximum of one across seven eligible squads. Restricting late eligibility to a single squad left total late exposure at three but forced that squad to receive all three. Its Gini is zero because there is only one eligible member, illustrating why the complete vector and cohort definition must accompany the coefficient.

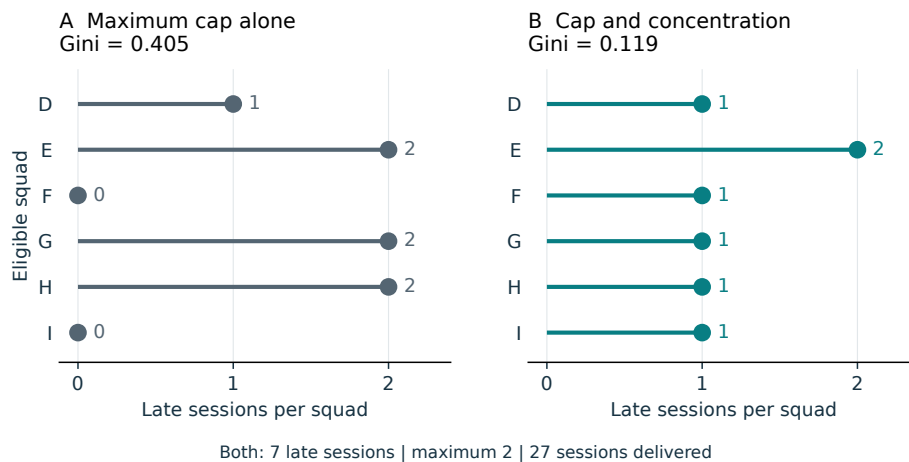


FIGURE 1. RESIDUAL INEQUALITY AFTER THE SAME MAXIMUM-EXPOSURE CAP. THE FOUR-CLOSURE WITNESSES BOTH ALLOCATE SEVEN LATE SESSIONS WITH A MAXIMUM OF TWO. THE LATE-COUNT VECTORS FOR D-I ARE (1, 2, 0, 2, 2, 0) UNDER THE MAXIMUM CAP AND (1, 2, 1, 1, 1, 1) AFTER PAIRWISE-DIFFERENCE MINIMIZATION.

TABLE 2. SPECIFIED SENSITIVITY SETTINGS, EACH USING THE INTACT PHYSICAL CALENDAR UNLESS INDICATED. ALL DISPLAYED EXPOSURE OBJECTIVES ARE SOLVER-CERTIFIED; THE FOUR-SESSION SETTING IS CERTIFIED INFEASIBLE.

Setting	Demand	Eligible n	Late Z^*	Max. M^*	Gini
Intact	27	6	3	1	0.500
One closure	27	6	4	1	0.333
Four closures	27	6	7	2	0.119
Eight squads	24	5	0	0	0.000
Ten squads	30	7	6	1	0.143
Two sessions	18	6	0	0	0.000
Five juniors	27	4	3	1	0.250
Eight juniors	27	1	3	3	0.000
Two late-only squads	27	6	6	3	0.667
Four sessions	36	6	Infeasible		

Each setting changes the stated element while retaining the other indexed restrictions. "Junior" denotes late-ineligible squads in these calculations. Gini values are descriptive within each eligible cohort and should not be ranked across different cohort definitions.

The late-only H/I setting demonstrates failure of aggregate bounds to characterize feasibility. Both squads required three late sessions, so $Z^* = 6$ rather than the physical-capacity lower bound of three, and $M^* = 3$ rather than the average-count lower bound of one. The optimized exposure vector was (0, 0, 0, 0, 3, 3), with $G = 0.667$. Four sessions per squad required 36 appointments against 32 available and was infeasible.

CYCLIC QUOTAS AND CUMULATIVE EXPOSURE

The cyclic-quota rule was feasible for the intact, one-closure and four-closure calendars and attained $T = 9, 8$ and 5 , respectively. In these three conditions, its quotas matched the minimum concentration. Appointment-level feasibility determines whether the counts can be assigned without violating court, coach, transport or daily-session restrictions. In the late-only H/I setting, the calculated quota was one for every eligible squad, which contradicted the required three late sessions for H and I; the feasibility check rejected it.

Alternating the two verified intact-calendar quota groups over six weeks gave all six eligible squads exactly three late appointments, while preserving three practices per squad per week. Cumulative Gini was therefore zero. Repeating the same three-squad group produced (6, 6, 6, 0, 0, 0) up to squad order, with cumulative $G = 0.500$. At odd weeks the alternating groups differed by one late appointment; at even weeks accumulated counts were equal (Figure 2). A total of 18 late sessions across six squads supplies the exact lower bound of three for the largest cumulative exposure, which the rotation attained.

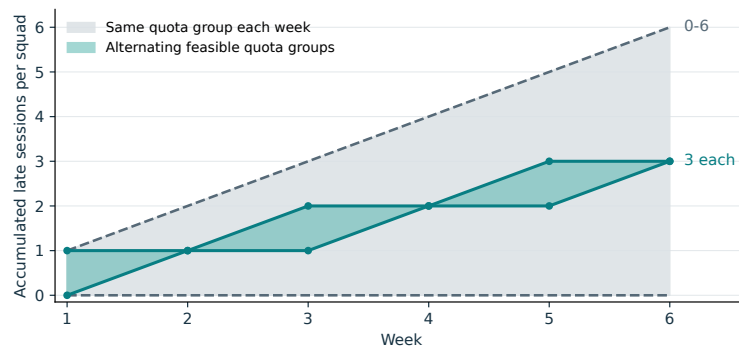


FIGURE 2. ACCUMULATED EXPOSURE OVER SIX REPETITIONS OF THE INTACT CALENDAR. THE PLOTTED BANDS COVER THE LEAST AND MOST EXPOSED ELIGIBLE SQUADS. REPEATING THE SAME QUOTA GROUP CONCENTRATES ALL LATE APPOINTMENTS IN THREE SQUADS; ALTERNATING TWO FEASIBLE GROUPS EQUALIZES COUNTS AT WEEKS 2, 4 AND 6. THE CALCULATION ASSUMES UNCHANGED WEEKLY AVAILABILITY.

DISCUSSION

The four-closure calendar distinguishes maximum protection from balance across all eligible squads. Removing four of the 24 non-late appointments forces seven of the 27 practices to begin at 20:30. Six eligible squads require a maximum of at least two, yet that cap permits both (1, 2, 0, 2, 2, 0) and (1, 2, 1, 1, 1, 1). Minimizing pairwise differences reduces T from 17 to 5 and Gini from 0.405 to 0.119 while preserving seven late starts and all 27 practices. The third objective therefore determines a part of the distribution left unresolved by the maximum (Ding and Balakrishnan, 2023).

Quota arithmetic succeeds when an appointment assignment satisfies the resulting counts. The intact, one-closure and four-closure calendars permit quotas differing by at most one late session. When H and I are available only at 20:30, however, each requires three late sessions. The equal-count quota of one contradicts that availability. The total rises to six and the maximum to three, even though the physical calendar still contains 24 non-late appointments. Court supply alone therefore cannot determine the late-session minimum; squad availability must enter the allocation constraints.

Balancing late appointments also changes start-time consistency. Under incumbent priority, I starts at 20:30 on all three practice days. The minimum summed span across squads is two bands, compared with five under intact balanced allocation. The three-band difference quantifies the additional within-squad variation required by the ordered objectives for this calendar. A club using these priorities would first retain the minimum late total and maximum protection, then minimize concentration, and finally select the smallest available span.

Repeated selection of the same weekly late group preserves inequality across weeks. Assigning all intact-calendar late appointments to D–F in each of six weeks gives accumulated counts (6, 6, 6, 0, 0, 0). Alternating D–F with G–I gives (3, 3, 3, 3, 3, 3) and a cumulative Gini coefficient of zero. Each displayed weekly timetable satisfies the same coach and transport restrictions. A booking ledger can record prior counts and use them when selecting the next quota group; changes in availability require the appointment constraints to be solved again.

The relevant athlete-level consequences depend on what follows the booking. A 22:00 finish leaves travel and recovery before the next school morning, but the calculation contains no return-home times, wake times, chronotypes or sleep recordings. Sleep and recovery guidance treats these circumstances as part of athlete preparation (Walsh et al., 2021; Kellmann et al., 2018). To assess the consequences of redistributed bookings, a club would need attendance records, travel times and sleep timing alongside the allocated appointments. Injury or retention outcomes would require longer observation.

Squad-level equality can also conceal differences in roster size and household transport. Research on community-sport spaces and sport leadership identifies organizational processes relevant to access (Jeanes et al., 2021; Burton, 2015). Before allocating appointments, managers should record mandatory junior exclusions, coach-sharing relationships, unavailable times and negotiable preferences separately. The resulting timetable can then show each squad's late count and the constraints responsible for any unequal allocation.

CONCLUSION

The 27-session calendar requires three late appointments with intact supply, four after one non-late closure and seven after the four specified closures. The smallest maximum among six eligible squads is one, one and two, respectively. Under four closures, balancing the remaining counts reduces Gini from 0.405 to 0.119 while retaining seven late appointments and all requested practices. Equal-count quotas attain these principal distributions when feasible, but fail when H and I require all three sessions at 20:30. Alternating two feasible weekly groups gives every eligible squad three late

appointments over six weeks. Court allocation should therefore specify total late demand, the maximum per squad, the full count distribution and cumulative exposure.

DECLARATIONS

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest related to this study.

DATA AVAILABILITY

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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