

CALIBRATION TRANSFER IN WEARABLE ACHILLES SHEAR-WAVE TENSIOMETRY

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ABSTRACT

To determine when a between-session change in Achilles tendon peak force or force impulse retains its direction under uncertain shear-wave calibration. Affine squared-speed calibration was combined with sagittal ankle moment balance. Both session-specific intercepts were retained. Exact inverse bounds were derived for multiplicative and additive transfer errors, and deterministic calculations used a first-session peak of 3000 N, transfer gain 0.90–1.10, force offset –200–200 N and a 0.6 s loading pulse. The study analysed mathematical force functions, without participant or instrument records. The transferred estimate obeys $\hat{F}_2 = \alpha F_2 + \beta$. At $M = 150$ N m, moment arms of 45–55 mm and non-Achilles moments of 0–15 N m admit tendon forces of 2454.55–3333.33 N. With the specified transfer limits, an estimated peak must fall below 2500 N to establish a decrease from 3000 N, or exceed 3500 N to establish an increase. Setting $\alpha = 1.08$ and $\beta = 150$ N converts a true peak of 2700 N to 3066 N, reversing the apparent direction. The same transfer changes the loading-pulse impulse from 990 to 1159.2 N s while preserving its normalised shape. A 2% multiplicative limit and 45 N additive limit bound relative force error by 5% at forces of at least 1500 N. A directional force conclusion requires a force-change interval that excludes zero. Peak force, accumulated impulse and normalised waveform shape require separate interpretation because an additive calibration error affects each quantity differently.

Keywords: Achilles tendon; wearable biomechanics; shear-wave tensiometry; calibration transfer; partial identification; tendon force impulse.

INTRODUCTION

Achilles shear-wave tensiometry estimates the speed of a mechanically induced transverse disturbance using accelerometers positioned along the tendon. The measured signal provides a route to internal loading assessment during walking, outdoor locomotion and running (Martin et al., 2018; Keuler et al., 2019; Harper et al., 2020, 2022; Reiter et al., 2024). For kinesiology, a between-session comparison asks whether a change in exercise or movement changes tendon force. Answering that question requires the force conversion to remain applicable after sensor removal and reattachment.

The conversion is usually expressed through an affine relationship between squared wave speed and tendon force or stress. Calibration experiments in isolated and anatomically retained tendons support this relationship over defined loading conditions (Martin et al., 2019, 2020). The slope and intercept depend on the physical interpretation of wave propagation: constitutive properties, fibre alignment and surrounding tissue alter the speed–stress relationship (Blank et al., 2022; Blank and Thelen, 2023). A fixed cross-sectional area or a repeatable propagation signal does not independently establish the force conversion.

Torque-based calibration introduces a second set of dependencies. Achilles moment arm varies with contraction state, measurement method and movement phase (Hashizume et al., 2014; Olszewski et al., 2015; Rasske et al., 2017). Holzer et al. demonstrated that the selected moment-arm function affects the magnitude and shape of torque-derived triceps surae force (Holzer et al., 2020). Net ankle torque also contains muscular and passive contributions outside the Achilles tendon. A close torque–squared-speed fit can therefore identify a combined coefficient without identifying Achilles force

alone.

This article derives the force intervals compatible with bounded calibration transfer and determines whether those intervals permit a strict increase or decrease. Session-specific intercepts are retained throughout. Peak force is evaluated against a first-session force interval; impulse is evaluated over an explicit duration; and waveform normalisation is examined separately. The numerical evaluation fixes every force, coefficient and uncertainty limit, so each conclusion can be checked directly from the equations.

METHODS

ANALYSIS DOMAIN AND NUMERICAL INPUTS

The study consists of algebraic derivations and deterministic evaluations of the resulting equations. All force values, wave coordinates and uncertainty limits are calculation inputs; no participant measurements or instrument records were analysed. Table 1 specifies the complete numerical domain. The intervals quantify the consequences of those selected limits and do not estimate the frequency of calibration error in human tendons.

The cited research concerns mechanical tensiometry, Achilles calibration, moment-arm measurement, tissue-wave mechanics and measurement reliability. Every reference falls between 1 January 2011 and 31 August 2024. The numerical limits in Table 1 are defined for this analysis rather than inferred as population ranges from the cited studies.

TABLE 1. NUMERICAL INPUTS USED IN THE FORCE, IMPULSE AND UNCERTAINTY CALCULATIONS.

Quantity	Value	Calculation
Net ankle moment M	150 N m	Mechanical force interval
Moment arm r	45–55 mm	Geometric uncertainty
Non-Achilles moment η	0–15 N m	Residual moment contribution
First-session peak P_1	3000 N	Direction of peak-force change
Transfer limits	$\alpha \in [0.90, 1.10]$; $\beta \in [-200, 200]$ N	Inverse peak and impulse bounds
Exact transfer	$\alpha = 1.08$; $\beta = 150$ N	Peak reversal and impulse distortion
Loading pulse	$300 + 2700 \sin^2(\pi t/0.6)$ N; $0 \leq t \leq 0.6$ s	Force impulse and normalised shape
Relative error limit	$u = 0.02$; $B = 0, 45, 150$ N	Load-dependent error envelope

WAVE MEASUREMENT AND LOCAL FORCE CALIBRATION

For sensor separation $d > 0$ and propagation delay $\tau > 0$, define wave speed $c = d/\tau$ and a centred squared-speed coordinate

$$q = c^2 - c_*^2, \quad F_j = a_j q_j + b_j, \quad a_j > 0. \quad (1)$$

The index j denotes a session. The fixed centring value c_* is a coordinate choice, not evidence of zero tendon load. If different centring values are used, their difference must be included in the subsequent signal offset. Here q has units $\text{m}^2 \text{s}^{-2}$, a has units $\text{N s}^2 \text{m}^{-2}$, and b has units N.

A tensioned-beam approximation motivates $c^2 \simeq (\sigma + k\mu)/\rho_{\text{eff}}$, with axial stress $\sigma = F/A$, cross-sectional area A , apparent shear modulus μ , shear correction factor k and effective inertial density ρ_{eff} (Martin et al., 2018, 2019). Over a suitable loading range, this gives $a \simeq A\rho_{\text{eff}}$. The intercept also includes the shear term and centring choice. Effective inertia, tissue structure and the observation method can affect the empirical relationship; a measured cross-sectional area alone therefore does not establish a or b (Keuler et al., 2019; Blank et al., 2022; Blank and Thelen, 2023). Eq. (1) is a local measurement model rather than a general constitutive law for a heterogeneous tendon.

INFORMATION SUPPLIED BY ANKLE TORQUE

Take plantarflexion as positive. The sagittal-plane net ankle moment is written

$$M = rF + \eta = r(aq + b) + \eta, \quad r > 0, \quad (2)$$

where r is the Achilles moment arm. The residual moment η combines non-Achilles plantarflexion, dorsiflexor opposition and any separately unaccounted passive contribution. It is positive when the former contributions exceed opposition. Triceps surae force sharing within the Achilles tendon is not an additional term in η ; individual soleus and gastrocnemius forces are outside this scalar model. The joint-axis definition and tendon line of action used for r must match the interpretation of M .

For constant r and $\eta = \eta_0 + \lambda q$, a perfect regression of M on q identifies

$$\kappa = ra + \lambda, \quad \zeta = rb + \eta_0. \quad (3)$$

For any selected $a' > 0$ and b' , the choices $\lambda' = \kappa - ra'$ and $\eta'_0 = \zeta - rb'$ give the same observed line. Anatomical or physiological restrictions may exclude some choices; the torque fit alone cannot select the tendon-force coefficients. Unknown r creates an additional scaling ambiguity even if $\eta = 0$ is imposed.

The covariance form separates moment-arm variation from effort-dependent residual torque. For ideal q , nonzero $\text{Var}(q)$ and a moment arm written $r = r_0 + \delta r$,

$$\kappa_q = r_0 a + \frac{\text{Cov}(q, \delta r(aq + b) + \eta)}{\text{Var}(q)}. \quad (4)$$

This is an identity for the population linear-projection slope, or for a finite dataset when covariance and variance use the same normalisation. It does not require a linear relationship between r and q . With constant r , it reduces to $ra + \text{Cov}(q, \eta) / \text{Var}(q)$. Reducing random error in the fitted slope does not remove the covariance term or identify its separate mechanical contributions.

If only ranges $r \in [r_L, r_U]$ and $\eta \in [\eta_L, \eta_U]$ are supplied, with $r_L > 0$ and $M - \eta_U \geq 0$, the compatible force interval is

$$\frac{M - \eta_U}{r_U} \leq F \leq \frac{M - \eta_L}{r_L}. \quad (5)$$

These limits are attained when all corner combinations are admissible. If geometry and residual moments are constrained jointly, the interval is an outer bound and the extrema should instead be evaluated over that joint admissible set.

SESSION-TO-SESSION FORCE TRANSFORMATION

Let the latent second-session coordinate be q_2 and its measured coordinate, expressed in the first-session convention, be

$$\tilde{q}_2 = gq_2 + h, \quad g > 0. \quad (6)$$

The gain acts on the *squared-speed coordinate*; it is not a percentage change in wave speed. The offset h has the same units as q . Using the first-session calibration without re-estimation gives

$$\begin{aligned} F_2 &= a_2 q_2 + b_2, & \hat{F}_2 &= a_1 (gq_2 + h) + b_1, \\ \hat{F}_2 &= \alpha F_2 + \beta, & \alpha &= \frac{a_1 g}{a_2}, \quad \beta = a_1 h + b_1 - \alpha b_2. \end{aligned} \quad (7)$$

The force transformation is completely specified by α and β when these coefficients are constant over the loading interval. Even if these two coefficients are identified against an independent force reference, g and a_2 remain confounded, and h, b_1, b_2 cannot be separated without additional observations. Attribution of drift to sensor placement or tissue change therefore requires more information than a force-level regression.

For $F_2 > 0$, the relative error is

$$\frac{\hat{F}_2 - F_2}{F_2} = \alpha - 1 + \frac{\beta}{F_2}. \quad (8)$$

If $|\alpha - 1| \leq u$ and $|\beta| \leq B$, a sufficient error bound is $u + B/F_2$. A chosen tolerance $\varepsilon > u$ over $F_2 \geq F_{\min} > 0$ requires

$$B \leq (\varepsilon - u)F_{\min}. \quad (9)$$

Here ε is the maximum accepted relative force error over the stated force range. Eq. (9) specifies the additive error allowed after the multiplicative contribution is allocated.

CERTIFYING THE DIRECTION OF A PEAK-FORCE CHANGE

For a common observation window and constant $\alpha > 0, \beta$, peak force satisfies $\hat{P}_2 = \alpha P_2 + \beta$. Let $y = \hat{P}_2$, with transfer bounds $0 < \alpha_L \leq \alpha \leq \alpha_U$ and $\beta_L \leq \beta \leq \beta_U$. If $y - \beta_U \geq 0$, the compatible peak-force interval is

$$L_2(y) = \frac{y - \beta_U}{\alpha_U}, \quad U_2(y) = \frac{y - \beta_L}{\alpha_L}. \quad (10)$$

For an independently established first-session peak $P_1 \in [L_1, U_1]$, the true change satisfies

$$P_2 - P_1 \in [L_2 - U_1, U_2 - L_1]. \quad (11)$$

A strictly positive lower endpoint certifies an increase; a strictly negative upper endpoint certifies a decrease. Otherwise the force-change interval includes zero, so no strict direction is identified. Equivalently,

$$\begin{aligned} \text{increase: } y &> \alpha_U U_1 + \beta_U, \\ \text{decrease: } y &< \alpha_L L_1 + \beta_L. \end{aligned} \quad (12)$$

An equality includes a possible zero change and does not certify a strict direction. If the first-session peak is itself a calibration-derived estimate, its force interval must first be obtained from its own calibration bounds; treating that estimate as an exact reference would understate uncertainty. Rectangular parameter ranges yield sharp limits when their corners are jointly feasible; correlated calibration uncertainty may permit narrower limits.

FORCE IMPULSE AND WAVEFORM INVARIANTS

For a fixed window of duration T , define force impulse $J_2 = \int_0^T F_2(t) dt$. Constant coefficients give

$$\hat{J}_2 = \alpha J_2 + \beta T, \quad J_2 \in \left[\frac{\hat{J}_2 - \beta_U T}{\alpha_U}, \frac{\hat{J}_2 - \beta_L T}{\alpha_L} \right], \quad (13)$$

provided the lower numerator is nonnegative. The change criterion follows Eq. (11) with first-session impulse limits. The additive uncertainty grows with T , so a peak-force error percentage cannot be assigned to impulse without checking the waveform mean and duration. Comparisons with different durations must retain their respective βT terms.

For a nonconstant waveform, positive affine transfer also gives

$$\frac{\hat{F}_2(t) - \min \hat{F}_2}{\max \hat{F}_2 - \min \hat{F}_2} = \frac{F_2(t) - \min F_2}{\max F_2 - \min F_2}. \quad (14)$$

Peak timing, within-window ordering and Pearson correlation with the true waveform are preserved; the correlation is exactly one. These properties are conditional on the same time base, constant coefficients and absence of additional filtering or random error. They do not establish absolute force or impulse agreement.

MEASUREMENT UNCERTAINTY AND REGRESSION ERROR

First-order propagation through $c = d/\tau$ gives

$$\frac{u_c^2}{c^2} \simeq \frac{u_d^2}{d^2} + \frac{u_\tau^2}{\tau^2} - \frac{2 \text{Cov}(d, \tau)}{d\tau}, \quad u_q^2 \simeq 4c^2 u_c^2, \quad (15)$$

where u denotes standard uncertainty and c_* is fixed. An uncertain centring value requires its own variance and covariance terms. For $F = aq + b$, the complete first-order form is $u_F^2 \simeq \mathbf{v} \Sigma \mathbf{v}^T$, where $\mathbf{v} = (q, a, 1)$ and Σ is the covariance matrix of (a, q, b) . Independence is a simplifying assumption, not a consequence of calibration.

Squaring also introduces bias: if an additive speed error e_c is zero-mean with variance u_c^2 , then $E[(c + e_c)^2] = c^2 + u_c^2$. Load-dependent speed variance can therefore create an offset that is not constant across the force range. Large relative delay errors invalidate the first-order approximation and require direct propagation through the nonlinear observation model.

For the restricted constant- r case, suppose the regression uses $x = q + e_q$, where e_q is zero-mean and independent of q and η . Independent zero-mean moment noise does not change the following population slope:

$$\kappa_x = \frac{ra \text{Var}(q) + \text{Cov}(q, \eta)}{\text{Var}(q) + \text{Var}(e_q)} = \kappa_q \frac{\text{Var}(q)}{\text{Var}(q) + \text{Var}(e_q)}. \quad (16)$$

This classical attenuation result is distinct from the mechanical ambiguity in Eq. (3). Correlated, biased or load-dependent measurement errors require a different model.

If an additional force-level error is bounded by $|e| \leq E$, so that $y = \alpha P_2 + \beta + e$, Eq. (10) becomes

$$L_2 = \frac{y - \beta_U - E}{\alpha_U}, \quad U_2 = \frac{y - \beta_L + E}{\alpha_L}, \quad (17)$$

under the corresponding nonnegative-numerator condition. A time-domain bound $|e(t)| \leq E$ also gives these peak limits and an additional impulse uncertainty of at most ET . Hard bounds and standard uncertainties must not be interchanged. A probability-based interval supports a directional conclusion only at its stated coverage; deterministic bounds require the specified limits to contain the actual errors.

NUMERICAL EVALUATION

The inverse peak bounds were evaluated at 801 equally spaced estimates from 2200 to 3800 N. Relative-error envelopes were evaluated at 801 forces from 500 to 4500 N, using $u = 0.02$ and additive limits $B = 0, 45, 150$ N. The loading pulse was evaluated at 1201 equally spaced points over $T = 0.6$ s. Its exact integral was checked against trapezoidal integration of those samples.

The nonzero-intercept transfer uses $a_1 = a_2 = 0.75 \text{ N s}^2 \text{ m}^{-2}$, $b_1 = b_2 = 600$ N, $g = 1.08$ and $h = 264 \text{ m}^2 \text{ s}^{-2}$. Eq. (7) then gives $\alpha = 1.08$ and $\beta = 150$ N. These parameters also define the four-load calculation in Section .

RESULTS

FORCE INTERVALS AND PEAK-CHANGE THRESHOLDS

Eq. (5) gives $F \in [2454.55, 3333.33] \text{ N}$ for the stated mechanical assumptions. The interval has width 878.79 N and contains every force compatible with the specified moment-arm and residual-moment limits. It is a deterministic interval, not a confidence interval. If $r = 50$ mm and $\eta = 10$ N m, the compatible force is 2800 N; dividing the same net moment by r while setting $\eta = 0$ gives 3000 N, an overestimate of 7.14%. If $\eta = -10$ N m instead, the true compatible force is 3200 N and the same shortcut underestimates it by 6.25%.

For the specified transfer rectangle, Eq. (12) gives a decrease threshold of 2500 N and an increase threshold of 3500 N. Values between or equal to these thresholds do not establish a strict direction against the first-session peak of 3000 N (Figure 1). For example, an estimated peak of 3066 N permits a true peak of 2605.45–3628.89 N. Comparing 3066 N directly with 3000 N gives an apparent increase of 2.2%, whereas the compatible interval includes both a decrease and an increase.

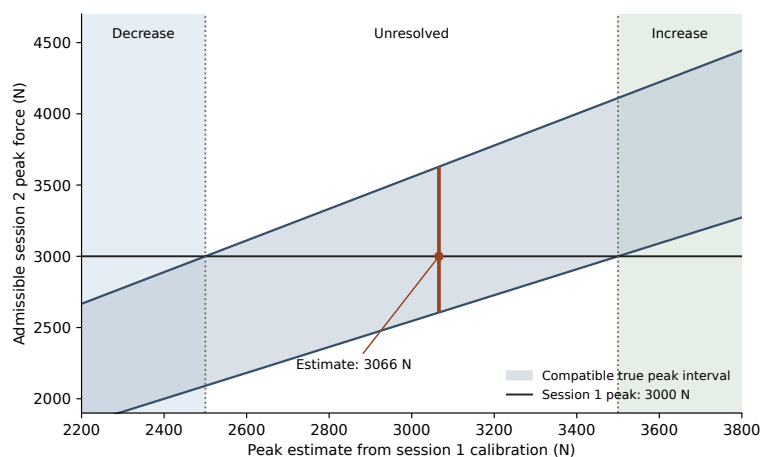


FIGURE 1. ADMISSIBLE SECOND-SESSION PEAK FORCE FOR $\alpha \in [0.90, 1.10]$ AND $\beta \in [-200, 200]$ N. THE HORIZONTAL LINE FIXES THE FIRST-SESSION PEAK AT 3000 N. THE VERTICAL LINES AT ESTIMATED PEAKS OF 2500 AND 3500 N DELIMIT THE REGION IN WHICH A STRICT DIRECTION CANNOT BE ESTABLISHED. AT AN ESTIMATE OF 3066 N, THE ADMISSIBLE FORCE INTERVAL IS 2605.45–3628.89 N. CURVES ARE CALCULATED FROM EQ. (10).

NONZERO INTERCEPTS AND LOAD-DEPENDENT DISTORTION

Table 2 evaluates four specified transfer conditions. With $\alpha = 1.08$ and $\beta = 150$ N, an unchanged peak of 3000 N is inferred as 3390 N, while an actual 10% reduction to 2700 N is inferred as 3066 N. The sign reversal persists with both calibration intercepts retained. Conversely, a lower multiplicative coefficient and negative aggregate offset produce a spurious decrease.

The additive-error term decreases as force increases (Figure 2). For $u = 0.02$ and $B = 45$ N, the bound is 11% at 500 N, 5% at 1500 N and 3.5% at 3000 N. Raising B to 150 N gives bounds of 32%, 12% and 7% at the same forces. The curves quantify the selected coefficient limits; instrument accuracy requires independent force measurements over the corresponding load range.

TABLE 2. EXACT TRANSFER CALCULATIONS RELATIVE TO AN INDEPENDENTLY KNOWN FIRST-SESSION PEAK OF 3000 N. ALL INPUTS ARE PRESCRIBED. THE LAST ROW CHANGES THE CALIBRATION SLOPE WHILE RETAINING THE NONZERO INTERCEPTS.

Transfer condition	True P_2 (N)	α	β (N)	Inferred peak/change
Unchanged force, positive transfer	3000	1.08	150	3390 N / +13.0%
True force decrease	2700	1.08	150	3066 N / +2.2%
Unchanged force, negative transfer	3000	0.92	-150	2610 N / -13.0%
Slope increase: $a_2 = 1.1a_1$	3000	10/11	600/11	2781.82 N / -7.27%

For the first two rows, $a_1 = a_2 = 0.75$, $b_1 = b_2 = 600$, $g = 1.08$ and $h = 264$ in the units defined in Methods. The third row uses $g = 0.92$, $h = -264$ with the same a , b . The last row uses $g = 1$, $h = 0$, $b_1 = b_2 = 600$. An aggregate β is not identical to the squared-speed offset h .

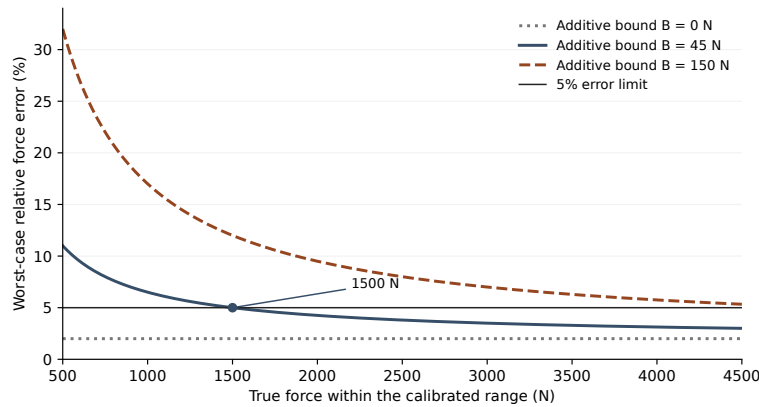


FIGURE 2. RELATIVE FORCE-ERROR LIMITS CALCULATED AS $100(0.02 + B/F)$ FOR $B = 0, 45, 150$ N. THE $B = 45$ N CURVE REACHES 5% AT 1500 N; THE $B = 150$ N CURVE REACHES THE SAME LIMIT AT 5000 N, BEYOND THE DISPLAYED FORCE RANGE. THE HORIZONTAL LINE MARKS THE SELECTED 5% TOLERANCE.

IMPULSE, SHAPE AND MEASUREMENT UNCERTAINTY

The prescribed pulse has peak 3000 N, mean force 1650 N and exact impulse $J = 990$ N s. Its cumulative impulse is

$$J(t) = 1650t - \frac{2700T}{4\pi} \sin\left(\frac{2\pi t}{T}\right). \quad (18)$$

Under the stated transfer, $\hat{J}(T) = 1.08(990) + 150(0.6) = 1159.2$ N s, an error of 17.09%, while peak error is 13% (Figure 3). The difference follows from the lower mean force in the impulse denominator, rather than from a change in timing. Min-max normalisation removes the entire constant affine distortion in this test.

With the broader transfer bounds used in Figure 1, the calculated impulse estimate 1159.2 N s admits a true impulse of 944.73–1421.33 N s. Against a first-session impulse of 990 N s, an increase is certified only above an estimated 1209 N s and a decrease only below 771 N s. The interval associated with 1159.2 N s contains 990 N s, so these limits do not establish an impulse increase.

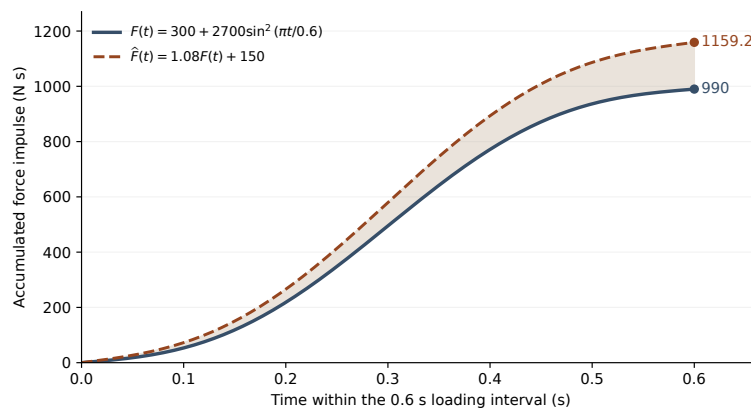


FIGURE 3. CUMULATIVE IMPULSE OF $F(t) = 300 + 2700 \sin^2(\pi t/0.6)$ N AND OF $\hat{F}(t) = 1.08F(t) + 150$ N OVER 0–0.6 s. THE TERMINAL VALUES ARE 990 AND 1159.2 N s. THE VERTICAL SEPARATION IS THE CUMULATIVE IMPULSE ERROR, $0.08J(t) + 150t$, AT EACH TIME.

For the spacing-delay calculation, set $d = 20$ mm, $\tau = 333.33 \mu\text{s}$, $u_d = 0.2$ mm and $u_\tau = 3.333 \mu\text{s}$, with independent uncertainties. At $c = 60 \text{ m s}^{-1}$, the first-order speed uncertainty is 0.849 m s^{-1} and the squared-speed uncertainty is $101.82 \text{ m}^2 \text{ s}^{-2}$. With fixed $c_* = 20 \text{ m s}^{-1}$, $a = 0.75$ and $b = 600$ in their defined units, $q = 3200 \text{ m}^2 \text{ s}^{-2}$, $F = 3000$ N

and the signal-only force uncertainty is 76.37 N (2.55%). This 76.37 N standard uncertainty includes only spacing and delay uncertainty. Calibration-coefficient covariance contributes the additional terms defined in Eq. (15) and the force covariance expression that follows it.

MULTI-LOAD IDENTIFICATION OF CALIBRATION TRANSFER

For the force relationship $F = 0.75q + 600$ N and fixed $c_* = 20 \text{ m s}^{-1}$, each selected force determines $q = (F - 600)/0.75$ and $c = \sqrt{q + 400}$. Table 3 evaluates these coordinates at 500, 1500, 3000 and 4500 N. The values are exact calculations under the stated relationship. The transfer $\hat{F} = 1.08F + 150$ N changes the four force estimates to 690, 1770, 3390 and 5010 N, respectively.

TABLE 3. FOUR-LOAD EVALUATION WITH $a_1 = a_2 = 0.75 \text{ N s}^2 \text{ m}^{-2}$, $b_1 = b_2 = 600$ N, $g = 1.08$, $h = 264 \text{ m}^2 \text{ s}^{-2}$ AND $c_* = 20 \text{ m s}^{-1}$. ALL VALUES ARE CALCULATED.

F (N)	q ($\text{m}^2 \text{ s}^{-2}$)	c (m s^{-1})	$\hat{F}$ (N)	Relative error (%)
500	-133.33	16.33	690	38.00
1500	1200.00	40.00	1770	18.00
3000	3200.00	60.00	3390	13.00
4500	5200.00	74.83	5010	11.33

Using the two endpoint force pairs gives

$$\alpha = \frac{5010 - 690}{4500 - 500} = 1.08, \quad (19)$$

$$\beta = 690 - 1.08(500) = 150 \text{ N}.$$

The two interior pairs satisfy the same line exactly. Their residuals, $\hat{F}_i - \alpha F_i - \beta$, are zero because they were evaluated from the assumed affine transfer. This calculation establishes identification within that model; it does not test the linearity of a physical instrument. Negative q at 500 N is admissible because q is centred, while $c^2 = 266.67 \text{ m}^2 \text{ s}^{-2}$ remains positive.

The additive force term explains the descending relative errors: $150/F$ contributes 30% at 500 N and 3.33% at 4500 N, on top of the constant 8% multiplicative contribution. Correlation between the four F values and their estimates is exactly one, even though their errors range from 11.33% to 38%. Correlation alone therefore does not identify acceptable force agreement.

For measured force pairs, independent axial-force calibration and its uncertainty are required to estimate α and β . Additional loads identify residual curvature and loading-unloading differences that two endpoints cannot reveal. Sensor spacing, orientation, contact and timing define the conditions under which the fitted coefficients apply. Net ankle torque can replace an axial-force reference only when the corresponding r and η are independently known or sufficiently bounded; matching torque alone does not identify matching Achilles force.

DISCUSSION

FORCE-CHANGE INTERPRETATION AFTER SENSOR REATTACHMENT

An estimated second-session peak above 3000 N is insufficient to establish an increase under $\alpha \in [0.90, 1.10]$ and $\beta \in [-200, 200]$ N. The estimate must exceed 3500 N because the maximum compatible transfer from an unchanged 3000 N peak is $1.10(3000) + 200 = 3500$ N. The decrease threshold follows from the opposite corner, $0.90(3000) - 200 = 2500$ N. These thresholds are consequences of the specified coefficient limits, rather than physiological cutoffs.

Both intercepts contribute to the force offset. With $b_1 = b_2 = 600$ N, $a_1 = a_2 = 0.75 \text{ N s}^2 \text{ m}^{-2}$, $g = 1.08$ and $h = 264 \text{ m}^2 \text{ s}^{-2}$, the aggregate offset is $0.75(264) + 600 - 1.08(600) = 150$ N. Assigning $\beta = a_1 h$ would instead give 198 N and would miscalculate the transferred force. An independent force regression can estimate α and β , but separating tissue calibration from sensor-coordinate gain and offset still requires additional physical measurements.

Moment-arm effects and non-Achilles moments are separate from signal-transfer error. The calibration work of Martin and colleagues concerns specified tissue and loading conditions (Martin et al., 2019, 2020); the moment-arm comparison of Holzer et al. concerns conversion from torque to muscle-tendon force (Holzer et al., 2020). The covariance term in Eq. (4) connects these mechanical dependencies to the fitted torque slope. Material and adjacent-tissue analyses also support retaining slope, intercept and load dependence in the wave relationship (Blank et al., 2022; Blank and Thelen, 2023).

WAVE-SPEED RELIABILITY AND FORCE AGREEMENT

Schneebeli et al. assessed Achilles wave-speed reliability in ten healthy participants using intraclass correlations, standard errors of measurement and minimal detectable changes (Schneebeli et al., 2022). Those quantities describe repeatability of the speed measurements under that protocol. Intraclass correlation depends on the selected model, definition and between-participant variability (Koo and Li, 2016); it does not establish an absolute force slope or intercept. Converting a detectable speed difference into a force difference requires the squared-speed transformation and calibration-coefficient uncertainty.

The 0.6 s pulse provides a direct distinction between agreement in shape and agreement in force. Positive affine transfer preserves peak timing and min-max normalised shape, yet changes the peak by 13% and the impulse by 17.09%. The greater impulse error arises because 150 N is divided by the mean force, 1650 N, rather than the peak, 3000 N. Using only waveform correlation or normalised curves would conceal both errors.

Neural-network prediction of Achilles stress uses wave-propagation features to estimate a calibrated loading target (Martin and Thelen, 2023). The target's mechanical assumptions remain relevant when interpreting its output. A torque-derived target does not independently remove uncertainty in moment arm or other joint moments. The patellar-tendon findings of Hahn et al. further demonstrate the importance of tissue- and contraction-specific assessment (Hahn et al., 2024); patellar accuracy cannot be assigned directly to Achilles measurements.

DOMAIN OF THE DERIVED BOUNDS

The force and impulse limits require constant positive α and constant β over the comparison window. Dispersion, load-dependent shear properties, motion artefacts, filtering and time-dependent contact can violate those conditions. Curvature in an independent force comparison requires bounds for the supported nonlinear relationship; time-varying transfer requires integrating its instantaneous error. Two force levels identify an affine pair but cannot establish that the affine description is adequate.

The ankle model uses a sagittal net moment and one Achilles line of action. It does not resolve subtendon loading, three-dimensional joint axes or distributed tendon stress. The selected uncertainty limits are not estimates of sensor-reattachment error in a human population. The spacing-delay result is a first-order standard uncertainty, whereas the peak-change thresholds use deterministic coefficient limits. A clinical or sporting force-change claim requires independent evidence that the applicable errors remain inside those limits.

CONCLUSION

For a first-session peak of 3000 N and transfer limits $\alpha \in [0.90, 1.10]$, $\beta \in [-200, 200]$ N, strict force-change direction is established only below an estimated 2500 N or above 3500 N. Within that interval, compatible second-session forces include an unchanged value. The transfer $\hat{F} = 1.08F + 150$ N turns a true 10% peak reduction into an apparent 2.2% increase and changes a 990 N s pulse impulse to 1159.2 N s. These exact calculations show why force magnitude, accumulated impulse and normalised shape require separate calibration checks. Independent force information, together with bounded ankle mechanics and measurement error, determines whether a between-session loading conclusion is supported.

DECLARATIONS

AUTHOR CONTRIBUTIONS

All authors contributed equally and all authors read and approved the final version of this manuscript.

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest related to this study.

DATA AVAILABILITY

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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