

BRAKING AND DEFENDER INTERCHANGEABILITY IN THREE-DUTY FUTSAL COVER ARRIVAL TIME, BODY ALLOCATION AND SIMULTANEOUS CLEARANCE

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ABSTRACT

To determine when four defenders can maintain pressure, interception and goal-side cover after one defender becomes unavailable. A theoretical calculation on a 40-by-20-m court combines rest-to-rest translation, heading adjustment and continuous-time separation checks. Defenders accelerate at 3.00 m s^{-2} , brake at 4.00 m s^{-2} and occupy three 0.20-m-radius duty regions by 1.80 s. Hall's matching condition identifies the number of complete defender losses compatible with cover. Two starting arrangements each generate six capability links and four complete assignments. Concentrated overlap requires defenders *A* and *B* for pressure and interception; distributed overlap permits cover after any single loss. The minimum neighbourhood surpluses are consequently zero and one. Minimum centre separations are 0.572659 and 0.662607 m against a 0.50-m requirement. Under independent defender availability of $q = 0.80$, complete-cover probabilities are 0.61440 and 0.81920. Central actions become eligible at 1.655625 s. At a 1.80-s deadline, their braking requirement is $2.807631 \text{ m s}^{-2}$. A ± 0.05 -s timing interval leaves the distributed arrangement's surplus between -1 and 1 at 1.70 s, but fixes it at 1 at 1.80 s. Defender interchangeability depends on the distinct bodies available to each group of simultaneous duties. An additional capability increases loss tolerance only when it adds a body to every duty group attaining the minimum surplus. The numerical values follow from the stated movement assumptions; they are not measurements of athletes.

Keywords: defensive cover; role flexibility; movement capability; concurrent duties; bipartite matching; futsal; collective coordination

INTRODUCTION

A defender who can pressure the ball carrier, close a passing lane and cover a receiver cannot occupy three separate regions simultaneously. Counting those actions as three individual capabilities leaves the allocation of bodies unresolved. In a four-defender unit, pressure and interception may depend on the same two defenders even when each duty has two alternatives. Loss of either defender then leaves two compulsory actions competing for one body.

Ecological dynamics locates tactical action in relations among performers, the ball, the goal and the constraints on movement (Vilar et al., 2012). Futsal coordination varies with ball and goal location (Travassos et al., 2012); attacker-defender relations also depend on the evolving competitive configuration (Vilar et al., 2012). Eligibility for a defensive duty therefore concerns an action from a particular starting position, velocity and heading before a particular deadline. A position label or a season-long skill rating does not supply that information.

Reciprocal compensation is central to team synergy theory (Araújo and Davids, 2016) and interpersonal coordination (Riley et al., 2011). Expert movement and neurobiological degeneracy permit different coordination solutions to preserve a function under changing constraints (Seifert et al., 2013, 2016). For simultaneous defensive duties, compensation additionally requires enough distinct eligible bodies. This requirement can be examined by connecting each defender to the duties that the defender can complete in time.

Bipartite assignment has a direct sporting precedent: Nouraie, Eslahchi and Baca use position-specific candidate groups and Hall's condition in football player selection (Nouraie et al., 2023). Laroche and colleagues relate bipartite surplus to the number of resource vertices removable while retaining a complete matching (Laroche et al., 2020). These established results provide the allocation criterion. The kinesiology question is how acceleration, braking, heading and route compatibility determine the links to which that criterion applies.

The calculation below holds the three duty regions and movement bounds constant while changing two defender starting positions. Both arrangements have six eligible actions and four complete assignments. Their difference lies in whether pressure and interception require the same two bodies. The analysis derives the arrival thresholds, checks simultaneous clearance and identifies the single added action that removes this dependence. Here, loss tolerance means preservation of all three duties after defender removal; psychosocial team resilience, as studied in elite sport (Morgan et al., 2013), concerns a separate construct.

DUTY REQUIREMENTS AND DEFENDER ELIGIBILITY

CONCURRENT DUTIES

Fix a starting instant t_0 , a short assessment interval and a non-empty set of $m \geq 1$ required defensive duties $\mathcal{T} = \{1, \dots, m\}$. Each duty has a success region, an arrival deadline h_j after t_0 , and any required orientation or holding condition. Let $\mathcal{P} = \{1, \dots, n\}$ contain the defenders eligible to act during that interval. A goalkeeper may be included only if the selected duties and legal actions permit it; the goalkeeper is not automatically counted as a spare outfield defender.

The duty set contains pressure on the carrier, occupation of an interception region and goal-side cover of a receiver only when the defensive instruction requires all three at the same instant. Separate duties require separate bodies. Mutually exclusive attacking continuations require separate assessments. If one defender can screen two threats from one region, that action constitutes a single duty or requires an allocation rule allowing one body to cover both threats. The number of duties is determined by the required actions and their spatial compatibility.

CAPABILITY LINKS

Let τ_{ij} be the shortest completion time of an admissible action by defender i for duty j , conditional on the starting position, velocity, orientation and specified opponent motion. An action is admissible only if it respects legal movement, attainable acceleration and braking, technical requirements and any holding condition. Set $\tau_{ij} = \infty$ when no such action exists. Define

$$C_{ij} = \mathbf{1}\{\tau_{ij} \leq h_j\}, \quad E = \{(i, j) : C_{ij} = 1\}. \quad (1)$$

The resulting bipartite graph is $G = (\mathcal{P}, \mathcal{T}, E)$. A link records the ability to complete an action from the starting state. Selection of that action is represented separately by the assignment variable. Observing one defender occupy a region does not establish the eligibility of the remaining defenders.

Acceleration and deceleration are substantial components of elite team-sport activity (Harper et al., 2019). Horizontal braking imposes biomechanical and neuromuscular demands that differ from those of forward acceleration (Harper et al., 2022). Change-of-direction assessment likewise separates turning performance from the linear-running contribution to total completion time (Nimphius et al., 2016). Eligibility must therefore include arrival, stopping and heading requirements. In the calculation below, these requirements enter explicitly through acceleration and braking bounds and a 10° heading tolerance.

DISTINCT-BODY ALLOCATION AND COMPATIBLE ROUTES

A complete assignment selects one eligible defender for each duty, without using a defender twice. With binary assignment variables z_{ij} , this requires

$$z_{ij} \leq C_{ij}, \quad \sum_{i=1}^n z_{ij} = 1 \text{ for every } j, \quad \sum_{j=1}^m z_{ij} \leq 1 \text{ for every } i. \quad (2)$$

The graph analysis assumes fixed duties, one distinct body per duty, and loss of a defender from every duty with the remaining capability links unchanged. Its mathematical guarantee concerns the existence of complete assignments. A physical guarantee additionally requires compatible actions for at least one assignment in every relevant remaining-player set. The interval includes arrival and the subsequent holding period.

If trajectories conflict, a mathematically valid assignment can be physically impossible. For known conflicts between assignments (i, j) and (k, ℓ) , an additional restriction $z_{ij} + z_{k\ell} \leq 1$ is needed. Such restrictions can destroy the sufficiency of ordinary matching. In that case, the matching calculation remains an optimistic feasibility bound until joint execution is checked. Similarly, a defender pulled toward the ball may retain one capability while losing another; that is an edge change rather than a complete player loss.

DISTINCT DEFENDERS AVAILABLE TO DUTY GROUPS

For a non-empty subset of duties $S \subseteq \mathcal{T}$, define its eligible-player neighbourhood

$$N_G(S) = \{i \in \mathcal{P} : C_{ij} = 1 \text{ for at least one } j \in S\}. \quad (3)$$

Each defender is counted once, even if that defender can perform several duties in S . The classical Hall condition states that a matching covering all duties exists exactly when

$$|N_G(S)| \geq |S| \quad \text{for every non-empty } S \subseteq \mathcal{T}. \quad (4)$$

An account of this theorem and matching theory is available in Diestel (2017). Each group of simultaneous duties must have at least as many eligible defenders as duties. Pressure and interception can each have two alternatives while sharing only two defenders; either defender's removal then violates Eq. (4) for that pair.

Define the *minimum neighbourhood surplus*

$$\rho(G) = \min_{\emptyset \neq S \subseteq \mathcal{T}} (|N_G(S)| - |S|). \quad (5)$$

This integer is measured in distinct defenders. A negative value means that complete cover is impossible under the stated assumptions. A value of zero means that complete cover exists but at least one duty subset has no spare eligible body. Positive values quantify a conditional guarantee against player losses. Because the full duty set is among the subsets, $\rho(G) \leq n - m$. No arrangement of four defenders can guarantee cover of three separate duties after two complete defender losses.

Proposition 1 (Guaranteed cover after defender losses). *For an integer $0 \leq r \leq n$, complete cover remains possible after every removal of at most r defenders, with all other links and duties fixed, if and only if $\rho(G) \geq r$.*

Proof. Removing a set U of at most r defenders leaves $|N_{G-U}(S)| \geq |N_G(S)| - r$. If $\rho(G) \geq r$, the remaining neighbourhood of every duty subset has at least $|S|$ members, so Hall's condition provides a complete assignment. Conversely, choose a subset S attaining the minimum. If $\rho(G) < 0$, cover already fails without removal. Otherwise, if $\rho(G) < r$, remove $\rho(G) + 1$ defenders from $N_G(S)$. This removal is possible because $|N_G(S)| = |S| + \rho(G)$ and S is non-empty. Fewer than $|S|$ eligible defenders remain for S , so no complete assignment exists. The removal uses at most r defenders. $\square$

Proposition 1 gives the Hall-condition proof of the matching-surplus interpretation (Laroche et al., 2020). The minimum is taken over duty groups because alternatives can overlap across actions. A *limiting duty subset* is any group attaining this minimum. Retaining its membership identifies the actions and defenders responsible for the loss tolerance. In the concentrated arrangement below, the sole limiting subset is pressure and interception, $\{a, b\}$, whose eligible defenders are $\{A, B\}$.

SIMULTANEOUS EXECUTION

Let $\mathcal{M}_U$ denote the complete matchings after removal of defenders U , and let $\mathcal{J}_U \subseteq \mathcal{M}_U$ contain those admitting legal, simultaneously executable trajectories and holding conditions. Physical cover requires

$$\mathcal{J}_U \neq \emptyset. \quad (6)$$

For an initially physically coverable arrangement, its physical loss tolerance cannot exceed $\rho(G)$. Equality holds exactly when Eq. (6) is satisfied for every $|U| \leq \rho(G)$. Assuming every complete matching executable is sufficient, but stronger than necessary. Certifying one full-team assignment does not certify the guarantee after all loss identities.

A shared passage can prevent simultaneous execution despite sufficient eligible defenders. Suppose three defenders are individually eligible for each of two duties, giving $\rho = 1$. Every individual action requires a shared one-person passage

for 0.60 s within a 1.00-s interval. Two concurrent duties require 1.20 s of passage occupancy, exceeding its capacity. Individual eligibility holds, but no complete assignment is jointly executable. The passage occupancy restriction must therefore accompany the distinct-body allocation.

COURT COORDINATES AND COMPLETION TIMES

THREE DUTY REGIONS AND FOUR STARTING POSITIONS

Consider a 40-by-20-m court with coordinates in metres, the defended goal at $(0, 10)$, and $t_0 = 0$. The ball carrier is fixed at $(8.7, 6.7)$; two potential receivers are fixed at $(15.3, 13.3)$ and $(16.7, 6.7)$. The three compulsory duties are occupation of a pressure zone centred at $a = (8, 7)$, an interception zone at $b = (12, 10)$ and a goal-side cover zone at $c = (16, 7)$. The interception centre lies on the carrier–first-receiver line. Each success region is a closed disk of radius $r_z = 0.20$ m. The tactical intention requires distinct defenders to occupy all three regions at the common deadline $h = 1.80$ s and retain them through $h + 0.20$ s. Completion consists of region occupancy, stopping and heading retention through 2.00 s.

Table 1 gives two starting arrangements. All defenders start at rest with heading 0 radians. The translational acceleration and braking bounds are $a_0 = 3.00$ and $b_0 = 4.00 \text{ m s}^{-2}$, with speed limited to $v_{\max} = 6.00 \text{ m s}^{-1}$. Translation is planar and isotropic. Heading can change concurrently at angular speed up to $\omega = \pi \text{ rad s}^{-1}$, independently of translation. These constants define the movement calculation and have not been estimated from athletes. Each duty requires the defender's centre to remain at rest within its disk and heading within $\varepsilon = 10^\circ$ of the centre-to-carrier direction. The target headings for a, b, c are approximately $-23.199^\circ, -135.000^\circ, -177.647^\circ$.

TABLE 1. DEFENDER STARTING COORDINATES IN METRES. ONLY THE POSITIONS OF A AND C DIFFER BETWEEN F AND R ; ALL DEFENDERS START AT REST WITH HEADING ZERO.

Defender	Arrangement F		Arrangement R	
	x	y	x	y
A	10.3	8.1	8.0	6.5
B	9.7	8.9	9.7	8.9
C	16.0	7.5	14.3	8.9
D	16.0	6.5	16.0	6.5

TRANSLATION, BRAKING AND HEADING TIME

For starting point p_i and duty centre q_j , the shortest distance to the success disk is

$$d_{ij} = \max\{\|q_j - p_i\| - r_z, 0\}. \quad (7)$$

For rest-to-rest motion along a free straight route, the minimum translation time is

$$T(d) = \begin{cases} \sqrt{2d(1/a_0 + 1/b_0)}, & d \leq d_*; \\ v_{\max}(1/a_0 + 1/b_0) \\ + (d - d_*)/v_{\max}, & d > d_*, \end{cases} \quad (8)$$

$$d_* = \frac{1}{2}v_{\max}^2(1/a_0 + 1/b_0).$$

The first branch follows by accelerating to speed v_p , then braking to rest: $d = v_p^2(1/a_0 + 1/b_0)/2$ and $T = v_p(1/a_0 + 1/b_0)$. The second branch adds travel at the speed cap. Here $d_* = 10.50$ m and all 24 player-duty distances across the two arrangements lie below that threshold; the largest calculated peak speed is 5.177 m s^{-1} .

Let θ_j be the required duty heading. The shortest angular change from initial heading θ_{i0} into its allowed interval is

$$\psi_{ij} = \max\{|\text{wrap}(\theta_j - \theta_{i0})| - \varepsilon, 0\}, \quad \tau_{ij}^0 = \max\{T(d_{ij}), \psi_{ij}/\omega\}, \quad (9)$$

where wrap gives the angle in $[-\pi, \pi]$. Independent translation and rotation make the slower requirement limiting. The required heading times are 0.073326, 0.694444 and 0.931371 s for a, b, c , respectively. Heading, rather than translation, limits the short $C_F \rightarrow c$ and $D \rightarrow c$ actions.

The free-route clock τ^0 is a lower bound on the admissible completion time τ : obstacle avoidance cannot shorten the distance or heading change. A clearance-certified straight route attains this bound, establishing equality for that pair. If $\tau^0 > h$, the pair is ineligible regardless of any detour. Table 2 provides all values needed to determine the binary graph at $h = 1.80$ s. Equality is certified for every eligible pair; values for ineligible pairs are lower bounds.

TABLE 2. COMPLETION TIMES FOR ELIGIBLE PAIRS AND LOWER BOUNDS FOR INELIGIBLE PAIRS AT $h = 1.80$ S. BOLD VALUES DENOTE ELIGIBLE PAIRS; $\geq$ IDENTIFIES A FREE-ROUTE LOWER BOUND. CLASSIFICATION USES UNROUNDED CALCULATIONS.

Defender	<i>F</i> : time (s)			<i>R</i> : time (s)		
	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>A</i>	1.656	1.656	≥ 2.557	0.592	≥ 2.443	≥ 3.020
<i>B</i>	1.656	1.656	≥ 2.728	1.656	1.656	≥ 2.728
<i>C</i>	≥ 3.020	≥ 2.296	0.931	≥ 2.728	1.656	1.656
<i>D</i>	≥ 3.020	≥ 2.443	0.931	≥ 3.020	≥ 2.443	0.931

JOINT ROUTES AND CLEARANCE

Each eligible defender translates toward the nearest point of the selected success disk. With unit route direction $e_{ij} = (q_j - p_i)/\|q_j - p_i\|$, its position is $p_i(t) = p_i + e_{ij}s_{ij}(t)$. For the triangular speed profile used here, write $t_a = v_p/a_0$ and $t_f = T(d_{ij})$. Then

$$s_{ij}(t) = \begin{cases} a_0 t^2/2, & 0 \leq t \leq t_a; \\ d_{ij} - b_0(t_f - t)^2/2, & t_a < t < t_f; \\ d_{ij}, & t \geq t_f. \end{cases} \quad (10)$$

Heading adjustment begins at t_0 and stops on entering the allowed interval. Early arrivals hold their positions and headings. An unassigned defender stays at the starting point. Each defender and opponent is represented by a disk of radius 0.25 m, requiring centre separation of at least 0.50 m throughout $[0, h + 0.20]$.

Every complete assignment listed below was checked against all other defenders, including the unassigned defender, and the three stationary opponents. On each interval between acceleration and braking breakpoints, relative position is quadratic in time, making squared separation a polynomial of degree at most four. Its minimum was found from interval endpoints and real roots of its derivative within that interval. This continuous-time check gives minimum separations of 0.572659 m in *F* and 0.662607 m in *R*, exceeding the 0.50-m requirement by 0.072659 and 0.162607 m, respectively. Consequently, all four complete matchings in each nominal graph admit the prescribed joint actions. Removing an unassigned defender retains those routes, certifying the relevant loss states. Figure 1 shows the geometry and one compensating assignment in *R*.

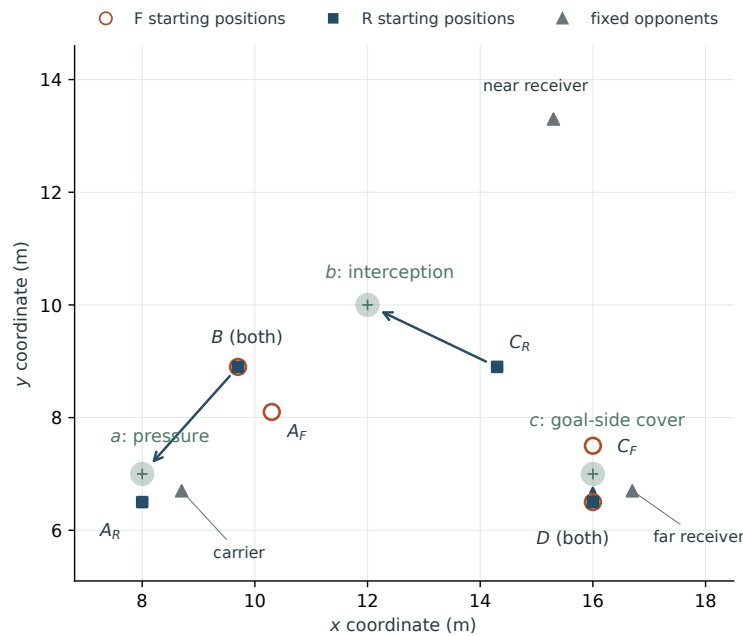


FIGURE 1. PRESSURE AT $a = (8, 7)$, INTERCEPTION AT $b = (12, 10)$ AND GOAL-SIDE COVER AT $c = (16, 7)$. OPEN CIRCLES MARK DEFENDER STARTS IN *F*; FILLED SQUARES MARK STARTS IN *R*. DEFENDERS *B* AND *D* SHARE THEIR COORDINATES ACROSS ARRANGEMENTS. EACH SHADED DUTY DISK HAS RADIUS 0.20 M. THE ARROWS GIVE THE *R* ASSIGNMENT $B \rightarrow a, C \rightarrow b, D \rightarrow c$ AFTER REMOVAL OF *A*. THE THREE OPPONENT POSITIONS REMAIN FIXED.

SIX ELIGIBLE ACTIONS WITH DIFFERENT LOSS TOLERANCE

The coordinates and certified times above yield the two four-defender, three-duty matrices in Table 3. Rows identify defenders and columns identify the pressure, interception and cover duties. Both matrices follow from the same movement bounds, duty regions and 1.80-s deadline.

TABLE 3. DEFENDER ELIGIBILITY AT 1.80 S. ENTRY 1 DENOTES A CLEARANCE-CERTIFIED ACTION MEETING THE ARRIVAL, STOPPING AND HEADING CONDITIONS; ENTRY 0 DENOTES A LOWER COMPLETION-TIME BOUND ABOVE THE DEADLINE.

	Concentrated overlap (F)				Distributed overlap (R)		
Defender	a	b	c		a	b	c
A	1	1	0		1	0	0
B	1	1	0		1	1	0
C	0	0	1		0	1	1
D	0	0	1		0	0	1
Duty degree	2	2	2		2	2	2

Both arrangements have six links. Each duty has two eligible defenders, and both have the same sorted player-degree distribution, $(2, 2, 1, 1)$. Both also have four complete assignments. In the order (a, b, c) , these are

$$\mathcal{M}_F = \{(A, B, C), (A, B, D), (B, A, C), (B, A, D)\},$$

$$\mathcal{M}_R = \{(A, B, C), (A, B, D), (A, C, D), (B, C, D)\}.$$

Thus even counting full assignments does not distinguish their loss tolerance.

Table 4 enumerates all seven non-empty duty groups. In F , $\{a, b\}$ has only $\{A, B\}$ available, giving surplus zero. Removing either defender leaves one body for two duties. In R , each group has at least one spare eligible defender, giving surplus one. After removal of A, B, C or D , respectively, R retains (B, C, D) , (A, C, D) , (A, B, D) or (A, B, C) in duty order (a, b, c) .

TABLE 4. ELIGIBLE DEFENDER SETS FOR EVERY DUTY GROUP AT 1.80 S. THE LAST TWO COLUMNS SUBTRACT THE NUMBER OF DUTIES FROM THE NUMBER OF ELIGIBLE DEFENDERS.

Duty group S	$N_F(S)$	$N_R(S)$	$ S $	F surplus	R surplus
$\{a\}$	$\{A, B\}$	$\{A, B\}$	1	1	1
$\{b\}$	$\{A, B\}$	$\{B, C\}$	1	1	1
$\{c\}$	$\{C, D\}$	$\{C, D\}$	1	1	1
$\{a, b\}$	$\{A, B\}$	$\{A, B, C\}$	2	0	1
$\{a, c\}$	$\{A, B, C, D\}$	$\{A, B, C, D\}$	2	2	2
$\{b, c\}$	$\{A, B, C, D\}$	$\{B, C, D\}$	2	2	1
$\{a, b, c\}$	$\{A, B, C, D\}$	$\{A, B, C, D\}$	3	1	1

The surviving-player sets permitting cover are ABC , ABD and $ABCD$ in F , and ABC , ABD , ACD , BCD and $ABCD$ in R . All eleven sets with fewer than three defenders fail in both arrangements. The only differences are ACD and BCD , each retaining just one of the pressure–interception pair A, B . Continuous-time clearance checks certify the listed assignments for every feasible surviving set.

AVAILABILITY AND COMPLETE-COVER PROBABILITY

Let each defender remain available with probability q , independently of the others. Complete cover in F requires both A and B and at least one of C, D . Complete cover in R requires any three defenders. Therefore

$$P_F(q) = q^2[1 - (1 - q)^2] = 2q^3 - q^4, \quad (11)$$

$$P_R(q) = \binom{4}{3} q^3(1 - q) + q^4 = 4q^3 - 3q^4. \quad (12)$$

Their difference is $2q^3(1 - q)$. It vanishes at $q = 0$ and $q = 1$, is positive between those endpoints, and reaches $27/128 \approx 0.21094$ at $q = 3/4$. This last value follows by differentiating the difference: its derivative is $2q^2(3 - 4q)$. These probabilities refer to complete occupancy of the three duty regions.

Independence is unnecessary for the qualitative ordering in this particular pair. Let W be the random surviving-player set under any joint availability distribution. The enumerated sets show that every state permitting cover in F also permits cover in R . Consequently,

$$\Pr(R \text{ covers}) - \Pr(F \text{ covers}) = \Pr(W = \{A, C, D\}) + \Pr(W = \{B, C, D\}) \geq 0. \quad (13)$$

TABLE 5. COMPLETE-COVER PROBABILITIES CALCULATED FROM EQS. (11) AND (12) AT FOUR VALUES OF INDEPENDENT DEFENDER AVAILABILITY.

Availability q	$P_F(q)$	$P_R(q)$	Difference
0.50	0.18750	0.31250	0.12500
0.75	0.52734	0.73828	0.21094
0.80	0.61440	0.81920	0.20480
1.00	1.00000	1.00000	0.00000

The ordering follows from inclusion of the feasible surviving sets of F in those of R . Comparing other arrangements requires their own surviving-set enumeration; the integer ρ alone does not determine probability.

Common-cause unavailability can remove the advantage. If all four defenders are available together with probability 0.8 and unavailable together with probability 0.2, each still has marginal availability 0.8, but both arrangements have cover probability 0.8. Marginal player probabilities therefore cannot identify cover probability without information about dependence. In sport, a shared movement requirement or a common positional displacement can also remove several capability links at once. Such events require the affected links and their joint occurrence probabilities to be specified.

DEADLINE AND BRAKING SENSITIVITY

For each shared central link, the distance to its success disk is $d_c = \sqrt{6.5} - 0.20 = 2.349510$ m. Translation limits the action, giving the exact threshold

$$h_c = \sqrt{7d_c/6} = 1.655625 \text{ s.} \quad (14)$$

Below h_c , the central duties cannot draw on those defenders. At or above it, the four central links enter in each arrangement: $A \rightarrow a, b$ and $B \rightarrow a, b$ in F ; $B \rightarrow a, b$ and $C \rightarrow b, c$ in R . Each spans $\sqrt{6.5}$ m from starting point to duty centre. For F , the next surplus-changing action is $C \rightarrow b$, with threshold

$$h_F = \sqrt{\frac{7}{6}(\sqrt{22.25} - 0.20)} = 2.295609 \text{ s.} \quad (15)$$

This route is clearance-feasible. It expands the neighbourhood of $\{a, b\}$ to $\{A, B, C\}$, raising $\rho(F)$ from zero to one. Write $h_a = T(0.30) = \sqrt{0.35} = 0.591608$ s and $h_o = \psi_c/\omega = 0.931371$ s (displayed to six decimals). For the displayed deadline range,

$$\rho(F; h) = \begin{cases} -3, & h < h_o; \\ -2, & h_o \leq h < h_c; \\ 0, & h_c \leq h < h_F; \\ 1, & h_F \leq h \leq 2.50, \end{cases} \quad \rho(R; h) = \begin{cases} -3, & h < h_a; \\ -2, & h_a \leq h < h_o; \\ -1, & h_o \leq h < h_c; \\ 1, & h_c \leq h \leq 2.50. \end{cases} \quad (16)$$

The thresholds h_a and h_o in Eq. (16) retain their exact defining expressions; numerical classification uses unrounded values. Once the surplus is one, the bound $n - m = 1$ proves that later links cannot increase it. The maximum surplus of one follows as soon as the clearance-certified links establish it. Negative surpluses indicate initial structural infeasibility, not negative numbers of tolerable losses.

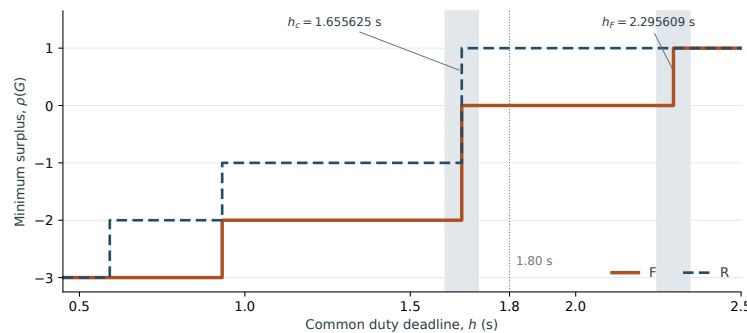


FIGURE 2. DEADLINE-DEPENDENT MINIMUM NEIGHBOURHOOD SURPLUS. AT $h_c = 1.655625$ S, FOUR CENTRAL LINKS ENTER EACH ARRANGEMENT AND ρ REACHES ZERO IN F AND ONE IN R . AT $h_F = 2.295609$ S, THE $C \rightarrow b$ LINK RAISES F TO ONE. SHADED STRIPS SPAN COMPLETION-TIME INTERVALS OF $h_c \pm 0.05$ S AND $h_F \pm 0.05$ S. LINKS ENTER AT EQUALITY UNDER $\tau \leq h_i$; INTERVAL WIDTHS ARE TIMING ASSUMPTIONS.

Changing braking magnitude also changes eligibility. Holding $a_0 = 3.00 \text{ m s}^{-2}$ and $h = 1.80$ s fixed, a central action is feasible only if

$$b_0 \geq \left(\frac{h^2}{2d_c} - \frac{1}{a_0} \right)^{-1} = 2.807631 \text{ m s}^{-2}, \quad (17)$$

provided the denominator is positive and the uncapped translation branch applies. At $b_0 = 2.50 \text{ m s}^{-2}$, the central clock exceeds 1.80 s, giving surpluses -2 in F and -1 in R . This threshold follows from the 2.349510-m travel distance and the 1.80-s deadline. A change in braking capacity can therefore remove the four central links while leaving the starting coordinates and duty regions fixed. Its numerical value is specific to those conditions.

ADDING INTERCEPTION CAPABILITY TO DEFENDER C

Adding $C \rightarrow b$ supplies a third body to the pressure–interception pair in F . Adding $A \rightarrow c$ supplies no additional body to that pair. The difference is expressed by the following single-link criterion, with all existing links retained.

Proposition 2 (The value of one new capability). *Let G' add one previously absent link (i, j) to G . Then $\rho(G')$ equals either $\rho(G)$ or $\rho(G) + 1$. It increases by one exactly when every limiting duty subset S contains j and has $i \notin N_G(S)$.*

Proof. For any duty subset S , adding (i, j) increases $|N_G(S)|$ by one precisely if $j \in S$ and $i \notin N_G(S)$; otherwise it leaves that neighbourhood unchanged. Thus every subset surplus increases by zero or one. The minimum cannot fall or rise by more than one. If any previously limiting subset remains unchanged, it preserves the old minimum. If every such subset increases, its surplus becomes $\rho(G) + 1$; all previously non-limiting surpluses were already at least $\rho(G) + 1$ because they are integers. The new minimum is then exactly $\rho(G) + 1$. $\square$

For arrangement F , adding C 's ability to complete duty b , as in the deadline calculation above, changes the neighbourhood of $\{a, b\}$ from $\{A, B\}$ to $\{A, B, C\}$. The single-duty and full-set surpluses remain at least one, so the guarantee rises from zero to one. By contrast, adding A 's ability to perform duty c increases the total link count but leaves the critical pair $\{a, b\}$ dependent on A, B ; the guarantee remains zero. Both changes add one capability, but only one supplies an additional distinct body to the limiting subset.

For the coordinates in Table 1, $C \rightarrow b$ becomes eligible when the deadline reaches 2.295609 s. At 1.80 s, shortening its completion time to the deadline would require a change in starting position or admissible movement. The calculation establishes the value of obtaining the link while leaving open the physiological or technical means of obtaining it. If a positional change removes another link, both changes must enter the eligibility matrix before loss tolerance is recalculated.

Once $\rho = n - m = 1$, four defenders cannot retain three separate duties after two complete losses. Further capability links can reduce travel time or provide additional assignments, but a second guaranteed loss requires a fifth eligible defender or fewer compulsory duties.

TIMING UNCERTAINTY AND OBSERVABLE COVER

COMPLETION-TIME INTERVALS

A completion time close to its deadline requires an interval assessment. If τ_{ij} is known only to lie in $[\ell_{ij}, u_{ij}]$, define two graphs:

$$C_{ij}^- = \mathbf{1}\{u_{ij} \leq h_j\}, \quad C_{ij}^+ = \mathbf{1}\{\ell_{ij} \leq h_j\}. \quad (18)$$

Provided those intervals contain the true admissible times and the other assumptions hold, $G^- \subseteq G \subseteq G^+$, which gives

$$\rho(G^-) \leq \rho(G) \leq \rho(G^+). \quad (19)$$

Agreement between the bounds identifies the structural surplus despite uncertain links. Disagreement identifies the need for additional measurement at the uncertain player–duty pairs. A collection of ordinary per-link confidence intervals is not automatically a simultaneous confidence statement about the entire graph; joint coverage must be addressed separately.

Position, speed and acceleration errors depend on the tracking technology and movement task (Linke et al., 2018). These errors can move an estimated completion time across its deadline. The interval in Eq. (18) must account for the errors relevant to the action; visual proximity alone does not establish eligibility.

For the numerical calculation, set $\delta = 0.05 \text{ s}$ around the clearance-certified completion times affecting the deadlines in Table 6. The central-link interval is $[1.605625, 1.705625] \text{ s}$, and the $C_F \rightarrow b$ interval is $[2.245609, 2.345609] \text{ s}$, with endpoints displayed to six decimals. Other pairs have lower bound $\tau^0 - \delta$ and upper bound infinity; their lower bounds exceed the tabulated deadlines. At 1.70 s, the central links may be eligible but are not guaranteed, giving R surplus bounds $[-1, 1]$. At 1.80 s, both bounds equal one. These are deterministic timing intervals, not sampling confidence intervals. Their graph bounds do not establish clearance for trajectories changed by timing variation.

TABLE 6. MINIMUM NEIGHBOURHOOD SURPLUS WITH $\delta = 0.05$ S AROUND THE CRITICAL COMPLETION TIMES. THE LOWER GRAPH USES UPPER TIME BOUNDS; THE UPPER GRAPH USES LOWER TIME BOUNDS. THE CENTRAL COLUMNS RETAIN THE EXACT NOMINAL CLOCKS.

h (s)	F			R		
	$\rho(G^-)$	$\rho(G)$	$\rho(G^+)$	$\rho(G^-)$	$\rho(G)$	$\rho(G^+)$
1.60	-2	-2	-2	-1	-1	-1
1.70	-2	0	0	-1	1	1
1.80	0	0	0	1	1	1
2.30	0	1	1	1	1	1

ARRIVAL, HOLDING AND ADJUSTMENT OBSERVATIONS

Complete cover has three directly observable requirements: one distinct defender in each 0.20-m-radius duty disk by the deadline, heading within 10° of the required direction, and position and heading retained for 0.20 s. Simultaneous execution adds a fourth requirement: every pair of body centres remains at least 0.50 m apart. The calculation evaluates all four under its deterministic motion laws.

Application to athletes requires completion-time observations from each defender–duty pair at the relevant starting position, velocity and heading. The admissible-time estimate must be obtained independently of the defensive episode used to assess compensation. Repeated actions supply within-defender timing variation; held-out repetitions assess whether the estimated interval classifies deadline eligibility correctly. The selected assignment, first arrival time, final heading and holding duration are separate observations.

Opposed defensive tasks must retain the information supplied by the ball, receivers and goal. Representative learning design (Pinder et al., 2011) and coordination analyses from dyads to teams (Santos et al., 2018) explain why removing those relations can change the action being tested. Futsal task constraints affect defensive behaviour (Pizarro et al., 2020); football practice can alter intra-team coupling and readjustment (Silva et al., 2016). Consequently, possession of an eligible compensating action and selection of that action must be assessed separately.

The deterministic comparisons identify three checks. At 1.80 s, R retains cover after each single removal, while F fails after removal of A or B . Below the 1.655625-s central threshold, the central links disappear. Adding $C \rightarrow b$ to F raises the surplus to one only if the other links remain available and at least one assignment remains jointly executable after each loss. In athlete observations, disagreement with these checks requires examination of the duty definition, completion-time estimate, route compatibility or adjustment delay. Repeated episodes are the observational units; successive video frames within an episode do not provide independent trials.

DEFENDER DEPENDENCE AND MOVEMENT RESTRICTIONS

The pressure–interception pair explains the difference between the arrangements. In F , all four complete assignments require both A and B . In R , the sequence of overlaps $\{A, B\}$ for pressure, $\{B, C\}$ for interception and $\{C, D\}$ for cover permits any one defender to be omitted. Equal link counts, duty degrees, player-degree distributions and assignment counts therefore leave loss tolerance unresolved. Table 4 identifies the bodies whose removal prevents cover.

This allocation property gives a discrete condition for reciprocal compensation (Araújo and Davids, 2016) and alternative functional organisation (Seifert et al., 2016). It concerns the availability of an assignment. Recognition, communication and adjustment can still delay its execution. Conversely, an uncovered region need not lead to a goal if the opponent does not exploit it. Complete region occupancy and competitive success remain different outcomes.

The movement laws restrict the scope of the numerical conclusions. Acceleration is isotropic, translation and heading control are independent, opponents are stationary and bodies are disks. Human acceleration and braking vary with direction, posture, fatigue, surface and technical action. Coupled turning and translation can increase completion time. The values 1.655625 s and $2.807631 \text{ m s}^{-2}$ consequently belong to the defined geometry and control bounds. They do not estimate thresholds for a player population.

Symmetry makes four links enter simultaneously at the central threshold. Defender-specific movement bounds would separate those arrival times. The interval calculation shows the consequence near a deadline: at 1.70 s, a nominal surplus of one coexists with a lower bound of -1 in R . At 1.80 s, the 0.05-s timing margin is sufficient to preserve the graph classification, although altered trajectories still require clearance checks.

Loss tolerance protects against every removal of up to ρ defenders, regardless of its frequency. Cover probability instead depends on the joint distribution of surviving players. Independent availability of 0.80 gives a difference of 0.20480 between the arrangements; perfectly shared availability gives equal probabilities of 0.80. Neither probability can be inferred from capability counts alone.

The three-duty requirement is also decisive. If one body can screen two threats, the allocation rule must allow that action. If the defensive instruction permits one region to remain uncovered, complete cover is no longer its success criterion. Longer possessions require changing positions, headings and deadlines to be evaluated at successive instants. Links feasible at different instants cannot be combined into one simultaneous assignment.

CONCLUSIONS

At a 1.80-s deadline, concentrated and distributed overlap provide six eligible actions and four complete assignments, yet tolerate zero and one arbitrary defender losses, respectively. Pressure and interception depend exclusively on A , B in the concentrated arrangement. Distributed overlap supplies three eligible bodies to each adjacent pair of duties and permits every single-loss state. Continuous-time separation checks confirm simultaneous execution under the stated motion laws. For fixed duties and unchanged links, the minimum neighbourhood surplus gives the exact assignment loss tolerance; physical cover additionally requires a jointly executable assignment for each relevant surviving set. An added action raises the surplus only when it supplies a new eligible body to every limiting duty group. Arrival deadlines, braking and timing intervals determine when those bodies enter or leave the eligible sets.

DECLARATIONS

AUTHOR CONTRIBUTIONS

All authors contributed equally and all authors read and approved the final version of this manuscript.

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest related to this study.

DATA AVAILABILITY

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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